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ESTIMATING THE PRESENT VALUE OF COMMON STOCKS BY THE VARIABLE RATE METHOD

W. Scott Bauman

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ESTIMATING THE PRESENT VALUE OF COMMON STOCKS BY THE VARIABLE RATE METHOD

A Study of the Present Value Theory and
a Practical Solution to the Problem
of Common Stock Valuation

W. Scott Bauman

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FOREWORD

The appearance in 1938 of John Burr Williams' book, *Theory of Investment Value,* popularized the present value theory of common stock evaluation. Until now, application of the theory to practical situations has been hampered by the fact that existing present value tables require their users to assume a constant rate of growth that will continue indefinitely. This assumption is recognized as being at variance with historical fact—that periods of faster-than-average growth are short and are sandwiched in between periods of normal growth.

Now, in this monograph, Dr. Bauman presents a series of variable rate tables which permit users to assume finite growth periods as short as two years or as long as thirty years. The availability of these new tables frees present value analysts from the unrealistic assumptions heretofore involved in practical applications of their theory.

Adherents of the present value theory should welcome Bauman's tables and his accompanying explanation of their construction and use.

Wilford J. Eiteman

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PREFACE

The substantial increase in common stock prices during the last ten years has brought investors to an impasse in their efforts to measure properly the value of equities by such orthodox standards as price-earnings ratios and dividend yield percentages. Investors are perplexed by a statistical paradox: during this period substantial investment profits as well as losses were realized from speculative stocks that were purchased at prices ranging from 30 to 100 times current earnings, the prices being such that current dividend yields were between zero and 2 per cent; moreover, only modest profits were realized from investments in many well-entrenched, high quality companies whose stocks were bought at prices between 12 and 16 times current earnings and showed current dividend yields of between 3 per cent and 5 per cent based on these prices.

In the last several years, attention has focused increasingly on the *present value theory* as a means of properly appraising the worth of common stock investments. The present value has gained wide acceptance as a sound concept; however, the development of a sound and practical method of applying this theory to actual investment situations presents a major problem.

This study develops a solution to this problem by the presentation of a useful method which may conveniently be applied by investors and others in making investment valuations.

I wish to thank the Ohio Citizens Trust Company of Toledo and the University of Toledo for support in the research effort. I am indebted to Professors Bruce H. Olson, H. H. Frisinger, Karl O. Mann, and John L. Mason and to Mr. Benedict J. Smith for helpful suggestions, to Professor Edward T. Kirkpatrick for computer programming assistance, and to R. J. Kriauciunas and T. T. O'Rourke for a considerable amount of research assistance.

W. Scott Bauman

The University of Toledo
Toledo, Ohio
July, 1963

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CHAPTER I

THE PRESENT VALUE THEORY

The present value concept is not new to the field of finance. It has long been an important tool in capital budgeting decisions and in the determination of life insurance premiums and annuity payments. Nor is the concept new to the field of investment, having been used for some time in computing payments for amortized real estate mortgages and in constructing bond tables which provide prices and yield rates to maturity. Although the concept is relatively new to the field of common stock investment, published works on the subject appeared more than twenty-five years ago.[1]

I shall not attempt to give an exhaustive description of the present value theory or a complete justification for its use in appraising common stocks; these matters have already been dealt with in existing literature.[2] The main purpose of this study is to develop an explicit and practical approach to be referred to as the *variable rate theory* and to describe the application of the *variable rate method* by the convenient use of prepared tables. Although other writers and investors have designed appraisal methods using the present value concept, most of these methods have at least one of three limitations: (1) they contain unrealistic assumptions; (2) they are too complicated and cumbersome for frequent practical use by investment decision-makers who are not statistical minded; (3) they are too narrow in scope to be readily applicable to many different classes of stocks. While the variable rate method presented below does not completely overcome these limitations, I believe it considerably minimizes them.

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[1] S. E. Guild. *Stock Growth and Discount Tables* (Boston: Financial Publishing Co., 1931); John B. Williams, *Theory of Investment Value* (Cambridge, Mass.: Harvard Univ. Press, 1938).

[2] A few citations are: George E. Bates, "Comprehensive Stock Value Tables," *Harvard Business Review*, XL (January-February, 1962), 53-67; O. K. Burrell, "A Mathematical Approach to Growth Stock Valuation," *The Financial Analysts Journal*, XVI (May, 1960), 69-76; John C. Clendenin and Maurice Van Cleave, "Growth and Common Stock Values," *The Journal of Finance*, IX, No. 4 (December, 1954), 365-76; Benjamin Graham, David L. Dodd, and Sidney Cottle, *Security Analysis* (4th ed.; New York: McGraw-Hill Book Co.), 1962, chap. 39; Philip Kotler, "Elements in a Theory of Growth Stock Valuation," *The Financial Analysts Journal*, XVIII (May-June, 1962), 35-44; Nicholas Molodovsky, "Stock Values and Stock Prices," *Ibid.*, XVI (May, 1960), 9-12, 79-92; Nicholas Molodovsky, "Valuation of Common Stocks," *Ibid.*, XV (February, 1959), 23-27; Robert M. Soldofsky, "Growth Yields," *Ibid.*, XVII (September-October, 1961), 43-47; and Charles Tatham, *Earnings Growth and Capital Appreciation*, issued by Bache & Co., in the year 1962.

The present value theory simply states that *the present worth, sometimes referred to as the intrinsic value, of a given investment, or stock, is equal to its future cash income, or dividends discounted at an appropriate yield rate or rates*. (The question of dividends versus earnings is discussed on p. 43.) In estimating the present worth of a given stock it is necessary to complete two steps: (1) estimate the future dividend income that the stock is most likely to produce; and (2) select a discount rate (or rates) most appropriate to the given investment. The discount rate is sometimes called the yield rate or capitalization rate and is simply the rate of return the investor expects to obtain on an investment if he invests an amount equal to the investment's present value.

The present value approach is beginning to revolutionize investment thinking for at least three reasons. First, the investment popularity of common stocks has increased rapidly among professional and institutional investors. These investors are able and willing to use sophisticated and elaborate analytical techniques, including the present value approach, in arriving at investment decisions.

Secondly, the fact that the earnings and dividends of many stocks are growing at widely divergent rates minimizes the usefulness of price-earnings ratios and dividend yields for valuation purposes. Let us assume, for example, that there are two stocks of the same quality, both currently earning $3.00 per share and paying $1.80 per share in dividends, and identical in every respect except that the earnings and dividends of the first stock (called stock X) are growing at 4 per cent per year, while those of the second stock (called stock Y) are growing at 15 per cent per year. If the market price of stock X is 45, it would have a price-earnings ratio of 15 and a dividend yield of 4 per cent. If stock Y trades at 75, it would sell at 25 times earnings and have a yield of 2.4 per cent. When we ask which of the two stocks is likely to provide the investor with the greatest return, it becomes quite apparent that the above ratio analysis is rather primitive. A valuation based on present earnings and dividends is static in nature, since it fails to reflect quantitatively the vital dimension of future changes in dividends. This is a serious shortcoming, since stock values are determined by future rather than by past earnings and dividends.

Thirdly, because of additional variables, the computations and construction of present value tables for common stocks are more difficult than for bond tables and annuity tables. The recent development of electronic computers, with their capacity to process larger quantities of data economically and rapidly, makes it easy to construct present value tables for common stocks. Indeed, electronic computers are already busily at work aiding investment analysts and investors in making ratio analyses and projections. The computer will never replace the investment analyst, but its use will be increasingly desirable for analysts.

CHAPTER II

THE VARIABLE RATE THEORY

A description of and the justification for the assumptions of the variable rate method are presented in this section. In estimating the present worth of a given stock, two steps are involved: first, estimating the size and the timing of future income, and, second, discounting this future income.

Estimating Future Dividend Income

The size of cash distributions paid by a business to its owners depends on several factors. The size of the distribution forecast in the short run for a company should be based on its established dividend policy, the current cash position, and its prospective cash sources and uses. In the long run, the dividend rate per share will depend on the size of future earnings (or cash flow) per share, the need for reinvestment of earnings, and the company's dividend policy.

Dividends based on earnings and cash flow

Over long periods of time there appears to be a very close positive relation between the trend of earnings and the trend of dividends for most publicly owned corporations.[1] Because significant changes in the general price level result in an understatement of depreciation charges on previously acquired fixed assets, and because significant changes in depreciation and income tax accounting methods have occurred since World War II, there may be merit in comparing dividend rates with cash earnings (earnings plus depreciation) per share.[2]

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[1] The earnings and dividends of all the stocks on the New York Stock Exchange from 1871 to 1938, as reported in a study by the Cowles Commission for Research in Economics, and the earnings and dividends of 50 "basic" stocks from 1937 to 1958 in a study by Molodovsky show that the long-term trends as a whole were almost identical for both earnings per share and dividends per share. See Molodovsky, "Valuation of Common Stocks," *The Financial Analysts Journal,* XV (February, 1959), 23-27.

Sometimes there is a lag between changes in earnings and changes in dividends for individual companies. If a firm's dollar volume of business expands rapidly, its working capital, fixed asset, and other disbursement requirements may be sufficiently great that its payout ratio (i.e., the ratio of dividends to earnings) declines. When the firm's growth in sales and earnings begins to level off, its dividend rate may continue to increase, with a subsequent increase in the payout ratio. Likewise, if earnings gradually decline, the dividend rate may, for a time, remain the same, or may decline proportionately less than the earnings reduction if the firm's internal cash needs also decline. In this instance, the payout ratio would rise.

In making an estimate of future dividends for a stock, it is therefore necessary (1) to examine the future earnings (or cash flow) prospects of the firm and (2) to exercise care in estimating the most plausible payout ratio. After reviewing the trend of earnings and payout ratios over the previous five to ten years, fitting these historical data into proper perspective with the present position and future prospects of the company, and adding a liberal helping of judgment and courage, the investor may forecast earnings and dividends per share.

High past rates of dividend growth

Much to the sorrow of many investors and investment analysts, the best earnings and dividend forecasts are not always those based on a simple extrapolation of past trends. This is particularly true of companies experiencing substantial rates of growth. Extrapolating high rates of growth defies the laws of nature, economics, and common sense. A small rapidly growing sapling does not eventually grow to the stratosphere. The laws of economics, which include the principles of diminishing returns, elasticity of demand, and economies of scale, place an upper limit on the growth rate of a firm. Business history serves also as a reminder that a company's earnings are affected not only by the ups and downs of the business cycle, but by life cycles of industries and corporations.

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[2] On a year-to-year basis, a much closer relation appears to exist between dividends and cash flow than between dividends and reported earnings during the 1948-62 period. See *The Outlook*, Standard & Poor's Corporation, XXXIV (December 24, 1962), 483-84.

As an up-and-coming young firm in a new industry expands from a small sales base, the growth rate of net sales may be very high, and the growth rate of earnings may frequently be higher than that of sales. In virtually every instance, high rates of growth eventually decline for one or more reasons. As the earnings base increases, the rate of growth can decline, even though earnings are increasing in larger absolute amounts. The process may be compared to the growth of a family. When parents have their first baby, the size of their family increases in that year by 50 per cent. If the parents have a child each year, the additions would increase the size of the family by 33 per cent in the second year, in the third year by 25 per cent, and in the fourth year by only 20 per cent. Although the annual rate of family growth is declining, we should not necessarily conclude that there is a decline in the parents' vitality.

As a company's sales volume increases, the market for the product may be approaching the saturation point. A highly profitable, rapidly growing industry not only invites rapid expansion of capacity by its existing competitors who want a larger percentage of the industry's market, but attracts outside firms who want to share in these high profits. Competition within the industry may tend to reduce the rate of growth in a firm's sales volume and, if severe price-cutting breaks out, may reduce net dollar sales (as happened in the transistor industry in 1960).

In making an earnings and dividend forecast for a company, it is important to estimate the probable position of the company in its life cycle. If a company has been experiencing a high rate of growth—and unless there is sufficient evidence to the contrary—the best earnings and dividend projection is probably one based on a decreasing rate of growth. Therefore, the most reasonable earnings and dividend forecast is possibly one based on a future trend (adjusted for cyclical fluctuations) that reflects an increase at a decreasing rate until the annual growth rate eventually approximates the secular growth rate for the majority of companies in the economy.

The transitional period

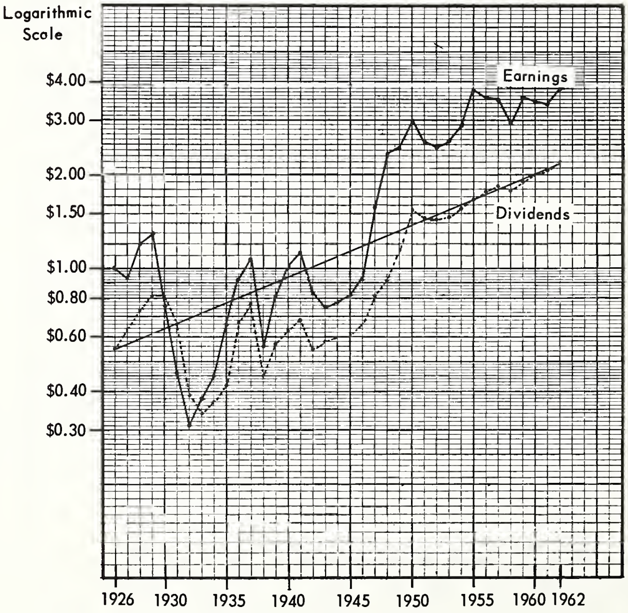
If the above pattern appears to fit the conditions of a company under study, then the next question is how soon this growth company will mature. In other words, how soon will the company's growth rate decrease to the growth rate of the majority of companies? The answer depends, of course, on a number of different factors. What phase of the growth cycle is the company in? Are earnings currently increasing at an increasing rate or are they starting to increase at a decreasing rate? Is the company strongly entrenched and well protected in its field? Or can its present position of leadership be rapidly duplicated or replaced by other companies? Is the company in an industry that has a barrier to entry by outsiders? Is the company on the threshold of exploiting many profitable innovations? Or is it, for example, in a rapidly growing industry with competitors conducting aggressive and successful research programs which may render the company's products technologically obsolete and hence expose the company to the loss of important markets? Is the demand for the product stable or rapidly shifting? Many of these forces are visible to the keen observer of particular industries and markets.

Once it is determined how well a given company is managed and protected, one may then estimate how fast or how slowly the company may be expected to mature in the future. If we estimate, for example, that a company will mature in fifteen years, we might assume that the present high rate of growth will decline gradually each year over the next fifteen years.

This is not to say that this will be the exact future growth pattern. We are saying that, for lack of better information, the most reasonable conclusion may be that the annual decline in the growth rate will be fairly constant during the transitional period of fifteen years, assuming that any cyclical deviations from this projected trend will be offsetting.

The rate of growth for mature companies

The next question is: What rate of growth in dividends should reasonably be expected for a company when it matures after its transitional period? If we are dealing with long periods of time, perhaps the best answer is found by examining the historic long-term growth rates for many companies. As illustrated in Figure 1, the rate of dividend growth of companies



Source: Standard & Poor's *Statistics*

Fig. 1. Dividends and earnings per share of the Standard & Poor's Industrial Stock Index, 1926-62.

TABLE 1

Earnings and Dividends per Share of the Standard & Poor's Industrial Stock Index, 1951-62

Year	Dividends	Earnings	Payout Ratio* (Per Cent)
1949-51 average	$1.37	$2.63	52
1951	1.45	2.55	57
1952	1.44	2.46	59
1953	1.47	2.59	57
1954	1.57	2.89	54
1955	1.68	3.78	44
1956	1.78	3.53	50
1957	1.84	3.50	53
1958	1.79	2.95	61
1959	1.90	3.53	54
1960	2.00	3.41	59
1961	2.08	3.37	62
1962	2.20	3.80†	58
1951-62 annual rate of growth	4.0%	3.1%	

*Dividends divided by Earnings.
†Preliminary.
Source: Standard & Poor's *Statistics*.

represented by the Standard & Poor's Industrial Stock Index over the 36-year period from 1926 to 1962 approximated 3.9 per cent per year, and the average rate of growth for earnings per share during this same period was 3.8 per cent per year. During the twelve years ending in 1962, as shown in Table 1, dividends for the Standard & Poor's Index grew at 4 per cent per year, very close to its 36-year rate; however, earnings per share increased during the 12-year period at only 3.1 per cent per year. While the 36-year rate of growth in earnings lagged only slightly behind the comparable growth in dividends, the 12-year growth in earnings was considerably below its corresponding 36-year growth trend as well as below the 12-year growth trend for dividends. As previously suggested, earnings reported by many corporations since World War II have been affected by certain distortions which have not affected cash flows. The most notable causes of distortions during this 12-year period were the Korean War 5-year amortization of war plants used in the early 1950's, accelerated rates of depreciation (double declining balance and sum of the digits depreciation methods) used since 1954, and revision of depreciation schedules and tax credits for business investments in 1962. Because of these distortions, cash earnings (net earnings plus depreciation and tax accruals) during the last twelve years have grown more rapidly than reported earnings.

For purposes of dividend projection, an annual growth rate of 4 per cent may provide a reasonable guide since it represents an approximate average rate of dividend growth for a representative group of stocks in recent years and over a long period of years, a period variously affected by depression, recession, inflation, peace, hot and cold wars, and prosperity. Since we are ignorant or uncertain concerning the future average growth rate of dividends of the majority of corporations, 4 per cent will be used as the standard, for lack of better evidence. Returning to our example once more, we may reasonably assume that after our company matures in fifteen years its dividends will grow thereafter at 4 per cent per annum.

We have now completed a framework for making reasonable estimates of future dividends. To summarize, the investor or the analyst of a given stock needs to answer two difficult questions: (1) What will be the rate of growth in dividends in the immediate future, and (2) at what time in the future will the company mature in such a way that its dividend growth will be typical of the majority of stocks?

Discounting Future Dividend Income

Once the future annual dividends are estimated, the second step is to select an appropriate discount rate or rates. The discount rate used for an investment should depend on the risks incurred or the rate of return that an investor desires or expects to obtain. If an investment appears to offer an unreasonably low return in view of the risks involved, then the investor will select an alternative investment that *does* appear to offer him a more reasonable return.

For very high quality stocks, the discount rate should be low; that is, if the estimated future income is fairly certain to be received, the investor should be satisfied with a modest rate of return. Likewise, for very speculative stocks, the discount rate should be high; that is, if the estimated income is subject to a high degree of uncertainty, the investor should seek a large potential rate of return.

If a stock is of high quality, an investor may be satisfied with perhaps a 6 per cent return. The return should not be much lower than this since higher quality bond investments are available at between 3 per cent and 5 per cent interest. If a stock is of low quality, an investor should expect a return of perhaps 10 per cent. The discount rate which applies to the income expected from a stock in the first year will be defined as the *initial discount rate*.

Size of discount rate a function of time

The accuracy of our forecasts diminishes in proportion to the futurity of the events we are making predictions about. The amount of income that an investor estimates he will receive from a stock in the distant future is subject to greater error and hence to more uncertainty than his estimates for the near future. Therefore, as an estimate of future income is made, the estimate for each succeeding year is subject to increased uncertainty.[3]

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[3] This point is taken by others. See Williams, *Theory of Investment Value*, pp. 59-60, 115-16. Kotler, "Elements in a Theory of Growth Stock Valuation," p. 42, says, ". . . for augmenting the discount rate over time is to reflect increasing riskiness of future earnings"; and p. 43, ". . . the investor becomes more uncertain of the level of actual earnings as the future recedes [i.e. the more distant future]." Note also Burke A. Parsons, *Common Stocks and Common Sense: A Note of the Valuation of Growth Stocks*, Baylor Business Studies, No. 54 (Waco, Tex.: School of

Therefore, if an investor estimates the dividends to be paid over the next hundred years, he should discount dividends to be paid in the distant future at higher rates than those for dividends to be paid in the near future. The higher discount rates, which apply to the income expected from a stock in the second and succeeding future years, will be defined as the *future discount rates*. Since the dividend estimate is subject to greater uncertainty for each succeeding future year, it follows that the future discount rate which is applied to each year's dividend should be higher than the rate applied to the dividends expected in the preceding year. The increase in each year's discount rate is a function, over time, of the increase in uncertainty.

Once the initial discount rate, based on the present quality of a stock, is determined, higher future discount rates are used simply because the future is more uncertain than the present. This relationship applies to low quality investments as well as to high quality ones. Forty years ago Pennsylvania Railroad was considered a high quality stock; at that time, however, it was next to impossible to foresee the company's predicament today.

John C. Clendenin states that the discount rate should perhaps not be increased over time since an investor may minimize the unforeseen risks that lie ahead by holding a diversified portfolio of stocks. Clendenin believes that by diversifying widely the investor has in the aggregate reduced the risk of loss from unforeseen developments which may affect individual companies and industries in the distant future.[4]

There may be some merit in this theory, but it has several limitations. First, *if* it is true that the aggregate risk of loss in a portfolio of stocks is less than the sum of the risk for each

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Business, Baylor Univ., 1962), p. 20, "... under the increasing uncertainty of the more remote future, the risk premium required ... [rises]."

[4] Conversations with Dr. Clendenin in December, 1960, and in August, 1962.

individual stock held, because unexpected losses will be offset by unexpected gains, then the future rate of growth in dividends for the average stock in the portfolio may tend to be mediocre, such as 4 per cent. While some stocks will have higher rates of dividend growth than anticipated, this may be offset by other stocks which will experience either lower than anticipated rates of growth in dividends or actual reductions in dividends. Therefore, if the holder of a diversified portfolio were to appraise each stock by discounting the more distant income at the same rate as the near-term income, then perhaps he should also project the future dividends of each stock at a modest rate of growth, such as 4 per cent. This solution would be unrealistic in valuing certain securities.

A second limitation of this argument is that only rarely are portfolios truly diversified against risk. Unexpected losses are not always offset by unexpected gains. Most investors who held widely diversified portfolios of stocks in the late 1920's uniformly experienced substantial reductions in dividend income in the 1930's. Many investors in diversified stocks likewise showed unsatisfactory results in the aggregate from 1946 to 1949. Also, most American portfolios are concentrated in United States securities. If we are discounting future income over the next hundred years, who can say that our economy will be as safe for investments in 2065 as in 1965? At the time when Rome was building a great empire, many Romans did not foresee the coming of the Dark Ages. The growth of government control and intervention within the United States and the rise of socialism and communism elsewhere in the last forty years increase the difficulty of predicting the future of the capitalistic system. The uncertainties of the remote future (which may take such catastrophic forms as depressions, financial panics, atomic wars, and major changes in the social, political, and economic structure) make it imperative that income projected for the distant future from a diversified portfolio be discounted at a higher rate than income projected for the near future.[5]

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[5] Clendenin recognizes that "there are some risks inherent in a possible change in the social milieu — taxes, socialism -- or in a decline

The same theory may be used to explain, in part, the existence of a positive sloping bond yield curve. Long-term government and corporate bonds usually provide higher yields than short-term issues since their holders are exposed to greater uncertainties over the life of the bonds than are the holders of short-term bonds. Consequently, one of the reasons why a yield curve has a positive slope is that the interest income and principal to be received in the distant future on a long-term bond are being discounted at a higher rate than is its near-term income.

The amount of increase in future discount rates

In discounting the future income of common stocks, if we use future discount rates that are higher than the initial discount rates, how much higher should they be? In answering this question it would be useful to know what discount rates have been implicitly used by most investors in the past. If we assume that market prices over a long period tend to reflect the investment valuation decisions of the majority of investors, then by determining the discount rates which related to these market prices we would know what discount rates were actually used by these investors.

Examination of the average dividend yield of the Standard & Poor's Industrial Index in the fourteen years from 1949 to 1962, as presented in Table 2, shows that the index had a median average yield of 4.13 per cent; stated differently, the average price in the 14-year period was 24.21 times its average dividend (i.e., the average price-dividend ratio). This might be considered a representative period. It includes years of pessimism, such as 1949-53, when market prices were relatively low and the stock index had an average current yield ranging from 8.28 per cent to 5.74 per cent, as well as years

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in the earning power of capital, to say nothing of a simple error in estimates, which diversification will not cure." Letter from Mr. Clendenin, dated September 4, 1963.

TABLE 2

Price, Dividends, and Yield of the Standard & Poor's Industrial Stock Index, 1949-62

Year	Average Price* (1941 to 1943 = 10)	Dividend	Average Yield† (Per Cent)
1949	$19.87	$1.14	5.74
1950	18.47	1.53	8.28
1951	22.59	1.45	6.42
1952	25.11	1.44	5.73
1953	24.84	1.47	5.92
1954	31.04	1.57	5.06
1955	42.60	1.68	3.94
1956	49.49	1.78	3.60
1957	42.61	1.84	4.32
1958	51.08	1.79	3.50
1959	61.17	1.90	3.11
1960	60.18	2.00	3.32
1961	68.28	2.08	3.05
1962	61.86	2.20	3.56
1949-62 median yield			4.13%
1949-62 median price-dividend ratio			24.21

*The average of the high and low price of the weekly index in each year.

†Dividend divided by Average Price.

Source: Standard & Poor's *Statistics*.

of optimism such as 1959-61, when market prices were relatively high, and the stock index had an average current yield varying between 3.05 per cent and 3.32 per cent.

If we assume, using the Standard & Poor's Index, that the average stock should normally have a current yield of about 4.13 per cent, we should normally expect a price-dividend ratio of about 24.2.[6] Therefore, if the average stock had a current dividend rate of $1.00 the stock would have a normal value or present value of about $24.20.

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[6] According to Molodovsky's article "Valuation of Common Stocks," the averate yield for the majority of stocks over the period 1871-1958 approximated 5 per cent, with a price-dividend ratio of 20. If economic conditions are more stable today and the business cycle is now less

As previously shown, the average annual rate of growth in the dividends of the stocks in the Standard & Poor's Index approximated 4 per cent. If we conclude from these studies that the average stock with a current dividend rate of $1.00 has a present value of about $24.20 and a dividend growth rate of 4 per cent per year, we have sufficient data to derive estimated discount rates for the average stock in the past. Table 3 shows the dollar amount of dividends, based on a growth rate of 4 per cent, which would be paid by this average stock in selected years.

If the initial discount rate depends on the degree of risk or uncertainty associated with the receipt of investment income in the immediate future, then it can be estimated after the investment quality of the average stock is determined. Examination of the quality of stocks used in the Standard & Poor's Index, shows that this group favored the larger companies in the medium to high quality category. Hence we may be willing to use an initial discount rate of perhaps $6\frac{1}{2}$ per cent.

We are now able to derive the estimated future discount rates by trial and error. After several trial runs, the future discounts presented in Table 4 were selected. The initial discount of 6.5 per cent is applied to the first year's dividend income of the average stock, described above; a future discount rate of 6.54875 per cent is applied to the second year's dividend income; and 6.59750 per cent is applied to the third year's dividend income, with each succeeding year's discount rate

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volatile than historically, so that business and financial risks are less for the majority of companies, then the price of the average stock may now justifiably be higher so that the average yield should come closer to 4 per cent than 5 per cent, as formerly. For the benefit of those readers who disagree with this last conclusion, a separate set of tables is presented in Appendix D; it is based on the long-term historical yield relationship (1871-1958) and has more conservative valuations (i.e., lower price-dividend ratios).

TABLE 3

Dividend Projection Used for the Standard & Poor's Industrial Stock Index

Future Year	Annual Dividend Growth Rate (Per Cent)	Annual Dividend Amount (assuming that last year's dividend rate was $1.00)
(Last year		1.00000)
1st	4	1.04000
2nd	4	1.08160
3rd	4	1.12486
4th	4	1.16985
5th	4	1.21665
. . . . . .	. . . . . .	. . . . . .
13th	4	1.66507
14th	4	1.73167
15th	4	1.80094
16th	4	1.87298
17th	4	1.94789
. . . . . .	. . . . . .	. . . . . .
98th	4	46.69457
99th	4	48.56235
100th	4	50.50485
101st	4	52.52504
101st	Future value of the stock index on the basis of 24.16 times its 101st year dividend.	$1,269.00497

TABLE 4

Discount Rates and Discount Factors Used for the Standard & Poor's Industrial Stock Index

Future Year	Discount Rates (i)	Discount Factors $(1 + i)^{-n}$	
1st	.0650000	$(1.0650000)^{-1}$	.93896714
2nd	.0654875	$(1.0654875)^{-2}$	.88085269
3rd	.0659750	$(1.0659750)^{-3}$	.82557963
4th	.0664625	$(1.0664625)^{-4}$	.77306798
5th	.0669500	$(1.0669500)^{-5}$	.72323550
. . . . . .	. . . .	. . . .	. . . .
13th	.0708500	$(1.0708500)^{-13}$	.41070297
14th	.0713375	$(1.0713375)^{-14}$	.38109377
15th	.0718250	$(1.0718250)^{-15}$	.35329864
16th	.0723125	$(1.0723125)^{-16}$	.32723387
17th	.0728000	$(1.0728000)^{-17}$	.30281763
. . . . . .	. . . . . .	. . . .	. . . .
98th	.1122875	$(1.1122875)^{-98}$	.00002956
99th	.1127750	$(1.1127750)^{-99}$	.00002545
100th	.1132625	$(1.1132625)^{-100}$	.00002189
101st	.1137500	$(1.1137500)^{-101}$	.00001881

being 0.04875 percentage points higher than that of the preceding year. The figure of 0.04875 per cent was computed by multiplying the initial discount rate of 6.5 per cent by three-fourths of 1 per cent.

Finally, we find that the present value of our average stock is $24.16. The computations are shown in Table 5. Column 1 shows the projected future dividend income based on an annual growth rate of 4 per cent; column 2 shows the yearly discount factors; and column 3 shows the future dividends discounted to their respective present values. The present values for dividends were computed for each year until individual present values (in col. 3) amounted to less than one-tenth of a cent; this occurred in the 101st year.

As shown in Table 5, the sum of these respective present values, including the present value of one share of stock in the 101st year, is equal to $24.16. Based on a current dividend rate of $1.00 the current yield is 4.14 per cent ($1.00 dividend by $24.16) and the price-dividend ratio is, of course, 24.16. These results come very close to the Standard & Poor's Index 1949-62 median average yield of 4.13 per cent and price-dividend ratio of 24.21 as shown in Table 2.

We have assumed that *if* the current dividend rate of the Standard & Poor's Index was $1.00 the present value of this index would be approximately $24.16. Since the current dividend rate for the index was actually $2.20 (1962), the approximate present value for the index is actually $53.15 ($2.20 times the price-dividend ratio of 24.16).

We have assumed that the amount of increase in the future discount rates of each succeeding future year is a function of increased uncertainty over future time. Thus the increase in the discount rates is also related to the size of the initial discount rate since the latter is related to the amount of uncertainty which exists in the first future year. In other words, if the future discount rates of a stock with an initial discount of 6.5 per cent increases 0.04875 percentage points each year (0.75 per cent of the initial discount rate of 6.5 per cent) then a very high quality stock with an initial discount of 5 per cent

TABLE 5

Present Values Computed for the Standard & Poor's Industrial Stock Index

Future Year	Future Yearly Dividends (As shown in column 2, Table 3)	Yearly Discount Factors (as shown in column 3, Table 4)	Present Values of the Future Yearly Dividends*
1st	$1.04000	.93896714	$0.97652
2nd	1.08160	.88085269	0.95273
3rd	1.12486	.82557963	0.92866
4th	1.16985	.77306798	0.90438
5th	1.21665	.72323550	0.87992
......	......	....	....
13th	1.66507	.41070297	0.68385
14th	1.73167	.38109377	0.65993
15th	1.80094	.35329864	0.63627
16th	1.87298	.32723387	0.61290
17th	1.94789	.30281763	0.58985
......	......	....	....
98th	46.69457	.00002956	0.00138
99th	48.56235	.00002545	0.00123
100th	50.50485	.00002189	0.00110
101st	52.52504	.00001881	0.00098
101st	1,269.00497 (1 share of stock)	.00001881	0.02387†

The sum of the present values $24.16 (4.14%)

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*Future Yearly Dividends multiplied by Yearly Discount Factors.

†A small point not mentioned previously is the value of one share of this stock as shown in Tables 3 and 5 for the end of the 101st year. As shown in Table 3, this value is expected to be $1,269. This figure was determined by multiplying the dividend estimated in the 101st year, which is expected to be $52.52504, by 24.16, the price-dividend ratio selected for this stock. As shown in column 3 of Table 5, the present value of this $1,269 stock is only $0.02387, or a little over two cents.

should have future discount rates increasing each year by 0.0375 percentage points (0.75 per cent of 5 per cent), and a speculative stock with an initial discount rate of 10 per cent should have future discount rates increasing each year by 0.075 percentage points (0.75 per cent of 10 per cent), etc. These discount rates are illustrated graphically in Figure 2.

The Present Value of Stocks with Average Growth

It was determined in the preceding section that a stock with an initial discount rate of 6.5 per cent and an annual dividend

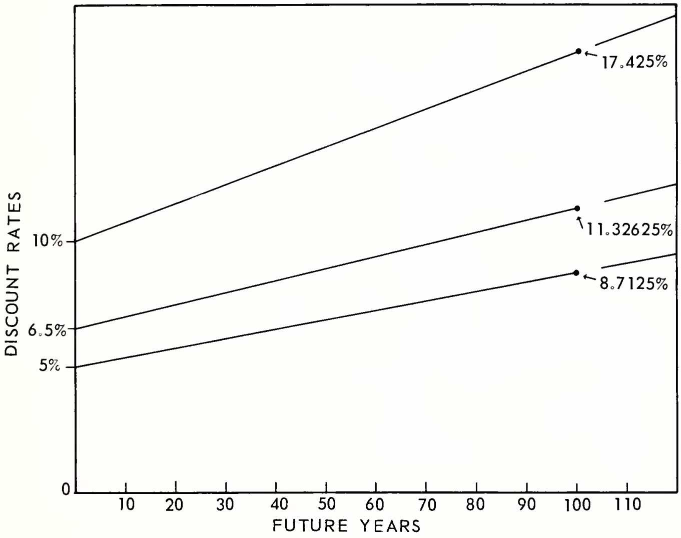

Fig. 2. Initial and future discount rates for a speculative stock, for a medium-to-high quality stock, and for a very high quality stock. The future discount rates increase by constant absolute amounts in a linear fashion. The size of these constant amounts is computed by taking 0.75 per cent of the respective initial discount rates. This study uses future discounts rates based on a linear function since this appears to be a reasonable assumption. Although three appears to be no evidence presently available to suggest that risks concerning the future arise in any other manner, it is possible that the future risks inherent in certain stocks increase at rates other than 0.75 per cent, or increase in other than a linear function, such as by an exponential function, etc.

growth rate of 4 per cent should have a present value equal to approximately 24.2 times its current dollar dividend rate. For reference purposes, this price-dividend ratio is shown in Table B-1 of Appendix B. The figure 24.2 appears under the columnar heading for the 6.5 per cent initial discount rate and on the line labeled for a 4 per cent growth rate. The price-dividend ratios of other stocks with a 4 per cent dividend growth rate are also shown on this line. Very high quality stocks with initial discount rates of 5 per cent have price-dividend ratios of 36.0; high quality stocks with initial discount rates of 6 per cent have ratios of 27.2; medium quality stocks with initial discount rates of 7 per cent have ratios of 21.7; and so on, with very speculative stocks with initial discount rates of 12 per cent having ratios of 10.7.

Conclusion

To summarize, we have just examined several long-term relationships of a stock which represents the average of a large group of stocks. It was concluded that during this historical period (1) the average price-dividend ratio was 24.2, so that for a stock with a current dividend rate of $1.00 the market price would be approximately $24.20; (2) the annual rate of growth in dividends for the average stock was 4 per cent; (3) the initial discount rate was 6.5 per cent, based on the investment quality of the average stock; and (4) the future discount rates were derived by increasing each succeeding year's rate so that the present value would amount to a factor of approximately 24.2.

We have devoted our attention to historical relationships between dividend growth rates and discount rates. While it is true that these relationships have existed for a long period, there is no assurance that they will continue to exist in the future. At present there appears to be little or no evidence to suggest that either future growth in corporate earnings and dividends or future investment uncertainties will be materially different from those of the past periods studied. Therefore, until such time as additional evidence is evaluated, these historical relationships seem to be the best available guides to the standards or yardsticks that can be used in making present value appraisals.

CHAPTER III

THE VARIABLE RATE METHOD AND STOCKS WITH HIGH GROWTH RATES

In the preceding chapter, the variable rate theory was discussed; in this section we will describe how to apply the variable rate method conveniently in solving investment valuation problems for growth stocks. Even though the reader may understand and agree with the theory as presented, he cannot easily apply these concepts in his appraisal work because of the considerable number of computations required to ascertain the present value in each stock analysis. However, if tables of values are prepared and made available to investors who appraise common stocks, much in the same manner as actuarially constructed bond tables are made available to bond investors, the task of stock investors will be reduced. The present value approach is not widely used today by investors because they are not willing to trap themselves in the cumbersome, unrealistic, or complicated formulas presented in other articles on this subject or to make time-consuming computations themselves. To meet these difficulties, a comprehensive set of tables is presented in Appendix B. Before referring to them, the reader should understand clearly the underlying assumptions used in their construction and know how to use the tables correctly.

To Find the Present Value

The best way to describe the assumptions underlying the tables is to give a simplified illustration of how to use the tables in finding the present value of a given stock. Three factors must be considered: the rate of dividend growth, the transitional period, and the initial discount rate.

Rate of dividend growth

Using International Business Machines Corporation as an illustration, we first make a forecast of the company's future dividend payments. The records, as presented in Table 6, show that dividends have grown at an average rate of 19.9 per cent per year over the last ten years, while earnings have increased 22.7 per cent per year. During the last five years, however, dividends have grown at over 25 per cent per year, while earnings have grown at slightly over 20 per cent per year.[1] The divergence in trends between dividends and earnings in recent years is reflected in a gradual increase in the dividend payout ratio. The dividend rate in 1962 was $3.00, while the annual dividend rate for 1963, based on the first quarter payment of $1.00, is indicated at $4.00. Dividends paid in the last twelve months (as of March 31, 1963) amounted to $3.25. In reviewing the company's financial position and assessing its prospects, we might conclude that its dividends will grow at the rate of about 25 per cent in the next twelve months (ending March 31, 1964).

The transitional period

International Business Machines has a strong, well-entrenched position in the rapidly growing electronic data processing industry. Because of the company's leadership and protected position in the industry, the investor might conclude that its business will not fully mature until fifteen years hence. Since it is assumed that the annual growth in dividends for the typical stock is approximately 4 per cent, the investor decides that IBM's dividends will grow at 4 per cent after it matures in the fifteenth year. Table 7, column 1, shows the assumed annual dividend growth rates for IBM in selected future years. Since the investor assumes that IBM will mature gradually over the next fifteen years, the annual dividend growth rate gradually

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[1] For a convenient method of finding approximate average annual compound rates of growth for 5- and 10-year periods, see Appendix A.

TABLE 6

Dividends and Earnings per Share and Payout Ratio for International Business Machines Corporation, 1951-62

Year	Dividends	Earnings*	Payout Ratio† (Per Cent)
1951-53 average	0.49	1.42	35
1953	0.51	1.59	32
1954	0.63	2.39	26
1955	0.66	2.93	23
1956	0.80	3.51	23
1957	0.97	4.05	24
1956-58 average	0.97	4.38	22
1958	1.13	5.57	20
1959	1.35	6.41	21
1960	2.00	7.45	27
1961	2.30	9.19	25
1962	3.00	10.97	27
(1963	4.00 indicated)		
10-year rate of growth (1953-62)‡	19.9%	22.7%	
5-year rate of growth (1958-62)§	25.3	20.2	
Last year's rate of growth (1962)	30.4	19.4	

*Includes undistributed earnings of foreign subsidiaries.
†Dividends divided by Earnings.
‡Using an average of 1951-53 data as a base.
§Using an average of 1956-58 data as a base.

TABLE 7

Dividend Projection Used for International Business Machines Corporation (1963) with a 15-year Transitional Period

Future Year	Annual Dividend Growth Rate (Per Cent)	Annual Dividend Amount (assuming the current dividend rate is $1.00)
(Last 12 months.		$ 1.00000)
1st	25.0	1.25000
2nd	23.6	1.54500
3rd	22.2	1.88799
4th	20.8	2.28069
5th	19.4	2.72314
. . . . .	. . . .	. . . .
13th	8.2	7.26730
14th	6.8	7.76148
15th	5.4	8.18060
16th	4.0	8.50782
17th	4.0	8.84813
. . . . .	. . . .	. . . .
140th	4.0	1,101.4165
141st	4.0	1,145.4731
142nd	4.0	1,191.2920
143rd	4.0	1,238.9436
143rd	Future value of the stock on the basis of 36.0 times its 143rd-year dividend	$81,741.9696

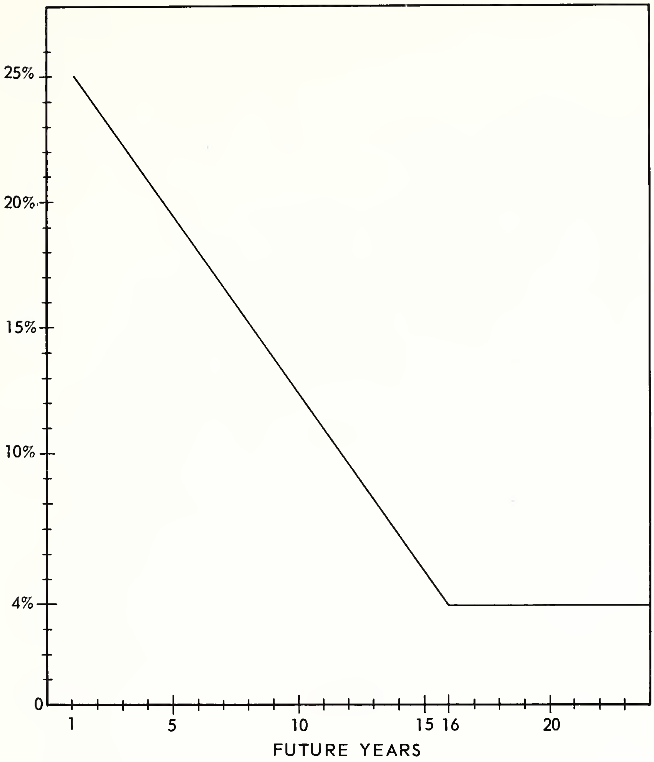

Fig. 3. Dividend projection used for International Business Machines Corporation (1963) with a 15-year transitional period.

declines from a 25 per cent rate in the first year to a 4 per cent rate in the sixteenth year and continues at 4 per cent thereafter. During the first fifteen years, each annual growth rate declines by a constant absolute amount (1.4 percentage points) from that of the preceding year.[2] If he finds no evidence to the contrary, the investor simply assumes that the growth rates during the 15-year *transitional period* decline in a linear fashion as shown in Figure 3. Using these growth rates, the estimated future dollar dividend payments are determined in column 2 of Table 7. The dollar figures are computed on the simplifying assumption that the annual dividend for the last twelve months was $1.00. Since the actual rate for IBM was $3.25, all the figures in column 2 should actually be three and one quarter times larger. The dividends are carried out to the 143rd year for reasons which will soon become obvious.

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[2] $\frac{25\%-4\%}{\text{16th year}-\text{first year}} = \frac{21\%}{15} = 1.4$ percentage points.

Initial discount rate

The final step is to select an initial discount rate reflecting the degree of uncertainty in income that the investor has estimated for IBM. Since the company appears well entrenched within its industry, has a strong financial position, is well managed, and seems likely to continue its steady and consistent record of sales, earnings, and dividends, the investor selects an initial discount rate of 5 per cent. Using the discussion of future discount rates in Chapter II, the investor increases each year's rate by 0.0375 percentage points, which is 0.75 per cent of the initial discount rate of 5 per cent. As noted in column 1 of Table 8, the discount rate in the first year is 5 per cent and increases by 0.0375 percentage points in each succeeding year. These discount rates are shown graphically in Figure 2, page 21.

In columns 2 and 3 of Table 8, these discount rates are expressed as compound discount factors. Note that the discount factor is quite small in the 140th year and decreases substantially in each succeeding year.

The present value

The future dollar income is shown in column 1 of Table 9, with discount factors in column 2 for selected years. Column 3 shows the future dividends discounted to their respective present values. The present values for dividends were computed for each year until individual present values (col. 3) amounted to

TABLE 8

Initial and Future Discount Factors Used for International Business Machines Corporation

Future Year	Discount Rates (i)	Discount Factors $(1 + i)^{-n}$	
1st (initial rate)	.050000	$(1.050000)^{-1}$	.95238095
2nd	.050375	$(1.050375)^{-2}$	.90638198
3rd	.050750	$(1.050750)^{-3}$	.86198923
4th	.051125	$(1.051125)^{-4}$	.81918612
5th	.051500	$(1.051500)^{-5}$	.77795360
. . . .	. . . . . .	. . . .	. . . .
13th	.054500	$(1.054500)^{-13}$	.50164283
14th	.054875	$(1.054875)^{-14}$	.47335418
15th	.055250	$(1.055250)^{-15}$	.44634408
16th	.055625	$(1.055625)^{-16}$	.42057702
17th	.056000	$(1.056000)^{-17}$	.39601680
. . . . . .	. . . . . .	. . . .	. . . .
140th	.102125	$(1.102125)^{-140}$	.00000122
141st	.102500	$(1.102500)^{-141}$	.00000106
142nd	.102875	$(1.102875)^{-142}$	.00000091
143rd	.103250	$(1.103250)^{-143}$	.00000079

TABLE 9

Present Values Computed for International Business Machines Corporation

Future Year	Future Yearly Dividends (as shown in column 2, Table 7)	Yearly Discount Factors (as shown in column 3, Table 8)	Present Values of Future Yearly Dividends*
1st	$1.25000	.95238095	$1.19047
2nd	1.54500	.90638198	1.40036
3rd	1.88799	.86198923	1.62742
4th	2.28064	.81918612	1.86831
5th	2.72314	.77795360	2.11848
......	....	....	....
13th	7.26730	.50164283	3.64559
14th	7.76148	.47335418	3.67392
15th	8.18060	.44634408	3.65136
16th	8.50782	.42057702	3.57819
17th	8.84813	.39601680	3.50401
......	....	....	....
140th	1,101.4165	.00000122	0.00134
141st	1,145.4731	.00000106	0.00121
142nd	1,191.2920	.00000091	0.00108
143rd	1,238.9436	.00000079	0.00097
143rd one share of stock	81,741.9696	.00000079	0.03492†
The sum of the present values			$142.26

*Future Yearly Dividends Multiplied by Yearly Discount Factors.

†As shown in Table 7, the normal value of one share of this stock at the end of the 143rd year is expected to be $81,741.97. This figure was determined by multiplying the dividend estimated in the 143rd year ($1,238.94) by 36.0, the price-dividend ratio for a very high quality stock with a growth rate of 4 per cent. This assumes that IBM will still be a high quality stock in the 143rd year. It is interesting to note that the present value of this large sum of $81,741.97 is only $0.03492, or a little over three cents, as shown in column 3 of Table 9.

less than one-tenth of a cent; this occurred in the 143rd year.[3] By adding the present values in column 3 of Table 9, we obtain \$142.26. This figure is the present value of all significant future dividend payments, including the present value of one share of stock in the 143rd year. Since this present value figure is based on a stock paying a current dividend of \$1.00 the figure of 142.26 is also the stock's normal price-dividend ratio *(price,* in this instance, meaning the stock's computed present value).

Since IBM paid dividends of \$3.25 in the last twelve months (through March 31, 1963) and not \$1.00, the investor would conclude that IBM's present value price should be approximately \$462.00.

$$\frac{\text{Present Value Price}}{\$3.25\ \text{Dividend}} = 142.3^{*}$$

$$\text{Present Value Price} = 142.3 \times \$3.25 = \$462.00.^{\dagger}$$

*Rounded to the nearest tenth.
†Rounded to the nearest dollar.

In arriving at this valuation, the investor made three assumptions: (1) the initial dividend growth rate will be 25 per cent; (2) the transitional period will be fifteen years; and (3) the initial discount rate should be 5 per cent. Proceeding on these assumptions, the investor may conveniently find the applicable price-dividend ratio by turning to Appendix B, Table B-8, which pertains to stocks with transitional periods of fifteen years, and following the line for an initial growth rate of 25 per cent across to the column headed 5 per cent. There he finds the price-dividend ratio, 142.3, which he should use.[4]

Since the annual dividend payments of many stocks change substantially from one year to the next, it may be more realistic in those instances to use as the current year's dividend payment a 3-year average of dividends paid in the preceding year, in the current year, and expected in the next year. In this way, we avoid computing present values that change substantially from one year to another, when such a change is attributable to a substantial change in a single year's dividend payments. (If we made this calculation for IBM we would have an average of $3.10, using $2.30 paid in 1961, $3.00 paid in 1962, and $4.00 expected in 1963.)

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[3] The duration of valuation significance for any given stock is dependent on (1) the current dividend rate, but most importantly on (2) the initial dividend growth rate, (3) the length of the transitional period, and (4) the discount rates. Most stocks have a period of significance of less than 120 years.

[4] For the equations used in the computations discussed in this and the preceding chapter, see Appendix C.

Since the market price on March 25, 1963, for IBM is $414.00, it can be concluded that other investors made assumptions about IBM on this date which differ from the explicit assumptions made by our investor. If our investor believes that his assumptions are correct, he will conclude that IBM's market price is approximately 10 per cent below the stock's present value. Shrewd or conservative investors seeking investments at prices substantially below their present values may reject the purchase of IBM because it is not sufficiently undervalued. Thus the investor seeks a large margin of safety which will allow for occasional errors of over-optimism.

To Find the Assumptions Based on the Market Price

Before the investor accepts IBM as being underpriced, he may wish at this point to recheck his assumptions and compare them with those made by investors in the market place. The market price of $414.00 suggests that other investors have made more conservative assumptions than he concerning one or more of three factors: (1) the stock is lower in quality so that it deserves a higher discount rate; (2) the dividend growth rate in the immediate future will be lower than 25 per cent; (3) stronger competition in its industry or some other factor may cause IBM to mature in *less* than fifteen years (i.e., IBM's dividends will begin to grow at only 4 per cent per year before the sixteenth year).

With a market price of $414.00 and a $3.25 dividend rate, IBM has a price-dividend ratio on March 25, 1963, of 127.4.

Examination of Table B-8, Appendix B, with some interpolation, shows that a price-dividend ratio of 127.4 could give us (1) an initial growth rate of 25 per cent with an initial discount rate above 5 per cent, falling between 5.0 per cent and 6.0 per cent; or (2) an initial discount rate of 5 per cent but with an initial growth rate less than 25 per cent, falling between 25 per cent and 20 per cent. Tables B-7 and B-8 in Appendix B show that a price-dividend ratio of 127.4 could give us (1) an initial growth rate of 25 per cent and an initial discount rate of 5 per cent, with a transitional period of less than fifteen years, falling between twelve and fifteen years; or (2) a combination of any two or three of the above assumptions.

If one concludes that IBM's present value is $462.00 he is accepting our investor's assumptions in preference to any of the above four. On the other hand, if one concludes that the present value is $414.00 he is accepting one of the four.

Summary

In this chapter we first showed how the investor used the variable rate tables to find the present value for a stock after explicitly formulating his own assumptions. Secondly, we showed how the tables could be used to aid the investor in identifying the assumptions, based on the current price, which other investors in the market place are using. Likewise, these tables can be used with stock indexes (or averages) such as the Standard & Poor's Stock Index and the Dow-Jones average, to help the investor (1) in determining whether the market appears high or low, and (2) in identifying the assumptions being used to justify the current market level. In those instances where the tables do not provide the exact rates or transitional periods the investor seeks, he may obtain approximations of them by interpolating the price-dividend ratios given. The most efficient way to apply the present value approach is by the use of tables such as Tables B-1 through B-11. In the preparation of such tables the computer is an invaluable tool. Once the computer program for this study was correctly designed the computer calculated and printed all the price-dividend ratios in Tables B-1 through

B-11 in approximately thirty hours.[5] Although we have included here only variable rate tables, these can easily be expanded to include additional values as well as different assumptions.

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[5] Larger computers could have done the job in less than five hours. With the enormous strides being made in computer technology and the rapid developments in quantitative analysis, statistical decision theory, operations research, etc., it is time for increased *practical* application of these tools to security analysis.

Lloyd S. Coughtry, Research Partner of Bear, Stearns, & Co., has made detailed 4-year earnings projections for nearly 100 leading electric utilities by the use of computer programming procedures. (See his article, "Computer—A Tool in Security Analysis," *Financial Analysts Journal,* XIX [January-February, 1963], 53-55.) Although more than 200 significant past and future items for each company were fed into the computer, the entire computational task for all the companies was completed in less than half an hour of computer time. He employed the FORTRAN programming language on an IBM computer, as was done in this study.

The Standard & Poor's Corporation has made extensive use of its computer in ratio analyses. The computer is also used to calculate past rates of earnings growth for 6,000 stocks. These figures form the basis of the corporation's report on the 200 fastest growing companies in the Standard & Poor's *Outlook.*

The University of Chicago has been conducting a computer study for Merrill Lynch, Pierce, Fenner, & Smith, Inc., in which investment rates of return are computed under varying assumptions for every stock listed on the New York Stock Exchange from 1926 to the present.

In an article entitled "A New Tool in Investment Decision-Making," *Financial Analysts Journal,* XIX (May-June, 1963), 55-62, Volkert S. Whitbeck and Manown Kisor, Jr., of the Bank of New York, describe interesting results obtained in valuing 135 stocks by using a multiple correlation of prospective earnings growth, earnings stability, and dividend pay-out ratios.

CHAPTER IV

THE VARIABLE RATE METHOD AND STOCKS WITH LOW GROWTH RATES

Just as some companies and industries experience temporarily high rates of dividend growth, others experience temporary reduction of their cash flows and consequently reduced ability to maintain or to increase dividend payments. A company may need to restrict dividend payments if general business conditions cause sales or earnings to drop because of a temporary decline in demand for its products or because of severe intra-industry competition. Likewise, a company undergoing a rapid expansion in sales may have such a heavy demand for reinvestment of funds that its cash dividend rate may remain unchanged for a period of years.

Temporarily depressed dividends

Therefore, when the annual dollar dividend rate of a stock is reduced or remains unchanged or has grown only slowly, the questions to be raised are: Will conditions probably change so that annual dividends will eventually increase at a growth rate approximating the rate for the majority of companies? And, if so, when in the future is the company's dividend record most likely to achieve this so-called normal growth rate?

The answer to the first question depends on whether the company's cash position will improve so that the management's dividend policy will be liberalized. In other words, are the factors contributing to a depressed or subnormal dividend growth rate temporary or permanent? This question is not always easy to answer. Investors frequently assume that when an industry is in a depressed or somewhat static situation it will remain there for an indefinite period. This was the attitude toward the tobacco industry in the middle 1950's and toward the utility industry in the 1940's. For example, many investors who had long accepted American Telephone and Telegraph Company's $9.00 dividend rate as a permanent fixture were taken by surprise when the stock was split three for one and the dividend rate was increased 10 per cent (on an adjusted basis) in 1959. Since the dividend rate was increased a second time in 1960, most investors now consider A. T. and T. to be a growth stock.

The transitional period

Once it is determined that the depressed dividend record for a given company is only temporary, one may assume that the company's former low dividend growth rate, lack of growth, or negative growth rate will change for the better. One then needs to estimate how much time will be required for the dividends to reach a normal growth rate.

If we estimate that the company's cash position and earnings trend will improve steadily, we might assume that the present low rate of growth will rise gradually by a constant amount each year over the next five years. This may not be the exact future growth pattern; it is simply the most accurate projection that can be made with the facts available.

To Find the Present Value

The Armco Steel Corporation provides a simple illustration of how the variable rate method may be applied to find the present value of a stock with a temporarily depressed growth rate.

Rate of dividend growth

First we estimate the company's future dividend payments. The record of dividends and earnings per share during the past ten years for Armco Steel is presented in Table 10. It will be

TABLE 10

Dividends and Earnings per Share and Payout Ratio for Armco Steel Corporation, 1951—62

Year	Dividends	Earnings	Payout Ratio* (Per Cent)
1951-53 average	1.50	3.20	47
1953	1.50	3.25	46
1954	1.50	3.93	38
1955	1.95	6.05	32
1956	2.55	6.03	42
1957	3.00	4.71	64
1956-58	2.85	4.88	51
1958	3.00	3.89	77
1959	3.00	5.21	58
1960	3.00	4.76	63
1961	3.00	3.89	77
1962	3.00	3.10	99
1963	2.50‖		
10-year rate of change (1953-62)†	7.2	-0.3	
5-year rate of change (1958-62)‡	1.0	-8.7	
Last year's rate of change (1962)§	0.0	-20.3	

*Dividends divided by Earnings.
†Using an average of 1951-53 data as a base.
‡Using an average of 1956-58 data as a base.
§Using 1961 as a base.
‖Conservative estimate.

seen that dividends during the last ten years had a steady but substantial average growth rate of 7.2 per cent per year, although in the last 5-year period dividends remained steady at $3.00 per year. The earnings trend was somewhat different. Earnings rose substantially in the early 1950's, topping out in 1955 and 1956, and subsequently fluctuated widely in a declining trend. This divergence in trend between earnings and dividends is shown in Table 10 by a rise in the payout ratio in the last six years. With the present very high payout ratio, the \$3.00 dividend rate is not well protected even though the cash flow per share is fairly large because of increased depreciation charges. Although it is likely that the current dividend rate of \$3.00 will be maintained in 1963, let us be conservative and use a dividend rate of \$2.50 for valuation purposes by the variable rate method. In reviewing the financial position and the outlook for Armco, the investor might conclude that the company will quite possibly pay \$2.50 in 1963. He assumes an initial dividend growth rate of 0 per cent (i.e., the dividend rate of \$2.50 assumed in 1963 will show no change from the dividend rate of \$2.50 that he is using for *variable rate* purposes).

The transitional period

Next the investor tries to determine how long it will take Armco's dividends to recover to an annual growth rate of 4 per cent. This will depend on several factors: the financial effects of Armco's investment in new, efficient labor-saving facilities, the prospect for future wage changes, the future needs of customers, foreign competition, and the industry's ability to break the politico-economic steel price barrier. Weighing these factors, the investor might conclude that it will take five years for the dividends to reach a normal growth rate.

Since it is assumed that the annual growth rate for dividends of the typical stock is approximately 4 per cent, the investor decides that Armco's dividends will grow at 4 per cent after the stock fully recovers at the end of the fifth year. Table 11, column 1, shows the assumed annual dividend growth rates for Armco in selected years. The annual growth rates gradually increase from a rate of 0 per cent in the first year to 4 per cent in the sixth year and continue at 4 per cent thereafter. During the first five years, each annual growth rate rises by a constant absolute amount (0.8 percentage points[1]) over that of the preceding year. If the investor finds no evidence to the

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[1] $\frac{4\% - 0\%}{\text{6th year} - \text{1st year}} = \frac{4\%}{5} = 0.8$ percentage points.

TABLE 11

Dividend Projection Used for Armco Steel Corporation (1963)

Future Year	Annual Dividend Growth Rate (Per Cent)	Annual Dividend Amount (assuming that the current dividend rate is $1.00)
(Last year		$ 1.00000)
1st	0.0	1.00000
2nd	0.8	1.00800
3rd	1.6	1.02412
4th	2.4	1.04870
5th	3.2	1.08226
6th	4.0	1.12555
7th	4.0	1.17057
......	...	....
97th	4.0	39.93928
98th	4.0	41.53685
99th	4.0	43.19833
100th	4.0	44.92626
100th	Future value of the stock on the basis of 24.2 times its 100th-year dividend	$1,087.21549

contrary, he simply assumes that the growth rates during the 5-year *transitional period* increase in a linear fashion as shown in Figure 4. The estimated future dollar dividend payments are determined by using these growth rates, as column 2 of Table 11 shows. The dollar figures are computed on the simplifying assumption that the annual dividend for the past year was $1.00. Since the *assumed* rate for Armco is $2.50, all the figures in column 2 should actually be two and one-half times larger than those shown. The dividends are carried out to the 100th year; beyond the 99th year the present value of individual dividends falls below one-tenth of a cent.

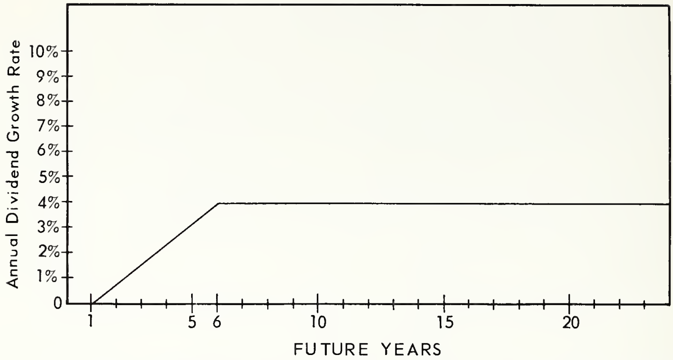

Fig. 4. Dividend projection used for Armco Steel Corporation (1963) with a 5-year transitional period.

Initial discount rate

The last step is for the investor to select an initial discount rate reflecting the degree of uncertainty in his estimate of the income from Armco stock. This large company is entrenched in a basic industry in the economy and appears reasonably well managed. It seems likely that sales and earnings will continue to fluctuate over moderately wide ranges reflecting cyclical changes in the demand for steel by durable goods and capital goods industries. Consequently the investor may conclude that Armco is a medium-to-high quality stock which deserves an initial discount rate of 6.5 per cent. Chapter II illustrated how each future year's discount rate is increased by 0.75 per cent of the initial discount rate of 6.5 per cent, or 0.04875 percentage points. These discount rate numbers are shown in Table 4, and illustrated graphically in Figure 2.

The present value

The future dividend income is shown again in column 1 of Table 12 and the discount factors for selected years in column 2. Column 3 shows the future dividends discounted to their present values. By adding the present values in column 3, we obtain $21.63. This figure is the present value of all significant future dividend payments, including the present value of one

TABLE 12

Present Values Computed for Armco Steel Corporation

Future Year	Future Yearly Dividends (as shown in column 2, Table 11)	Yearly Discount Factors (as shown in column 3, Table 4)	Present Values of Future Yearly Dividends*
1st	$1.00000	.93896714	$0.93896
2nd	1.00800	.88085269	0.88789
3rd	1.02412	.82557963	0.84549
4th	1.04870	.77306798	0.81072
5th	1.08226	.72323550	0.78273
6th	1.12555	.67599782	0.76087
7th	1.17057	.63126949	0.73895
......	....	....	......
97th	39.93928	.00003431	0.00137
98th	41.53685	.00002956	0.00122
99th	43.19833	.00002545	0.00109
100th	44.92626	.00002189	0.00098
100th (one share of stock)	1,087.2154	.00002188	0.02372
The sum of the present values			$21.63

*Future Yearly Dividends multiplied by Yearly Discount Factors.

share of stock in the 100th year. Since this present value figure is based on a stock paying a current dividend of $1.00, the figure of 21.63 is also the stock's normal price-dividend ratio (*price*, in this instance, meaning the stock's computed present value).

Since our conservative investor is using a $2.50 dividend rate for Armco and not $1.00 he would conclude that Armco's present value should be approximately $54.00.

$$\frac{\text{Present Value Price}}{\text{\$2.50 Dividend}} = 21.6 \text{ (rounded to the nearest tenth)}$$

$$\text{Present Value Price} = 21.6 \times \$2.50 = \$54.00.$$

In arriving at this valuation, the investor made three assumptions: (1) The initial dividend growth rate will be 0 per cent; (2) the transitional period will be five years, and (3) the initial discount rate should be 6.5 per cent. Proceeding on these assumptions, the investor may conveniently find the applicable price-dividend ratio by turning to Appendix B, Table B-3, which pertains to stocks with transitional periods of five years, and following the line for an initial growth rate of 0 per cent, across to the column headed 6.5 per cent. There he finds the price-dividend ratio, 21.6, which he should use.

Since the annual dividend payments for many cyclical stocks such as Armco have been known to change substantially from one year to the next, it may be more realistic to use as the current dividend payment a 3-year average: dividends paid in 1961 ($3.00), in 1962 ($3.00), and expected in 1963 ($2.50). In this way, we avoid computing present values that change substantially from one year to another when such a change is attributable to a substantial change in a single year's dividend payments. If we made this calculation for Armco we would have an average dividend of $2.83 and a subsequent present value of $61.00 (21.6 × $2.83).

As the preceding chapter explains, the investor can use the tables in Appendix B to identify the combinations of assumptions being used implicitly by investors at the current market price of $54.00 (March 25, 1963). If we use $2.50, for example, as our current dividend rate, then Armco's current price-dividend ratio is 21.6 ($54.00 ÷ $2.50). By looking in the tables for figures close to 21.6, the investor is able to identify other possible assumptions which are being made in the market place, as was done for IBM in Chapter III.

CHAPTER V

THE VARIABLE RATE METHOD APPLIED

The purpose of this chapter is to present evidence in support of the hypothesis that (1) a positive relation exists between dividend growth and investor profit, and (2) the present value theory and the variable rate method represent a sound approach and a very practical way of achieving the investor's profit objectives. The study is based on empirical evidence gathered from thirty stocks and covering the 10-year period from 1952 to 1961.

Earnings versus Dividends

Much recent literature indicates that the basis of common stock values is the size, stability, and rate of growth of corporate earnings and earnings per share.[1] Many investors believe that current and future earnings, in and of themselves, are the fundamental determinants of value. Such investors prefer to give primary attention to price-earnings ratios and the outlook for earnings per share. Of secondary or minor importance are dividend rates, payout ratios, and yield percentages. Although the modern investor seeking capital gains grants that an important relation usually exists between earnings and dividends, he nevertheless tends to think that the future *earnings* stream is more important than the future *dividend* stream.

Such thinking leads some to conclude that future dividend payments are of little or no consequence in the determination of stock values. This conclusion is, of course, in direct contradiction to the present value theory. Investors who make appraisals based primarily on earnings sometimes argue that a valuation method which relies on future dividends understates the value of companies which retain and profitably reinvest sizable amounts of earnings. On the contrary, the investor who makes a valuation based on future dividends must consider the effects of profitable reinvestment of earnings in his forecast of the future growth of earnings and dividends. For example, if we have two stocks of the same over-all investment quality, the first of which has a higher dividend growth rate than the second, then the first stock will be assigned a higher price-dividend ratio. The point is not that earnings should be disregarded but rather that future dividends should be regarded as well.

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[1] Douglas A. Hayes, *Investments: Analysis and Management* (New York: The Macmillan Co., 1961), pp. 284-86.

Dividend Growth and Investor Profits

The dividend growth and investor profits of thirty stocks[2] for the 10-year period 1952-61 are presented in Table 13. Dividend growth is defined as the average annual (compound) rate of dividend growth during the 10-year period. The stocks are listed in five groups based on a descending order of dividend growth, starting with 17.4 per cent for International Business Machines and ending with 0 per cent for Sinclair Oil.

Profit index

A profit index is presented in the second column of Table 13. This index, expressed in dollars, represents the market value of an investment on December 31, 1961, for an investor who owned $100.00 worth of the investment on December 31, 1951; in addition, it assumes that the investor reinvested all annual cash dividends in additional shares on December 31 of each year. The figures reflect all stock dividends and splits; however, taxes and brokerage expenses are disregarded. An

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[2] These were thirty of the most popular stocks (by dollar value) among over 225 investment companies in 1957. *Guide to Investment Company Portfolios* (Huntington, N. Y.: Vickers Associates, Inc., 1958).

TABLE 13

Average Annual Dividend Growth Rate and Profit Index for 30 Selected Stocks, 1952–61

Company	Average Dividend Growth Rate (Per Cent)	Profit Index
Group I		
International Business Machines Corp.	17.4	2,391
Minnesota Mining & Manufacturing Co.	14.3	953
Texas Utilities Co.	10.8	770
Bethlehem Steel Corp.	9.1	573
Gulf Oil Corp.	8.9	431
National Lead Co.	8.8	482
Group II		
Central & South West Corp.	8.5	703
Texaco Inc.	8.0	522
Goodyear Tire & Rubber Co.	7.9	1,015
United States Gypsum Co.	7.9	717
DuPont (E.I.) de Nemours & Co.	7.9	365
United States Steel Corp.	7.2	615
Group III		
Armco Steel Corp.	7.2	609
General Electric Co.	7.2	515
Merck & Co., Inc.	7.2	360
General Motors Corp.	6.5	535
General Public Utilities Corp.	6.4	493
Firestone Tire & Rubber Co.	6.3	567
Group IV		
Union Carbide Corp.	6.0	257
Amerada Petroleum Corp.	6.0	186
Goodrich (B. F.) Co.	5.8	314
Standard Oil Co. of Calif.	5.4	369
Standard Oil Co. (N. J.)	5.2	307
Shell Oil Co.	5.1	342
Group V		
Socony Mobil Oil Co., Inc.	4.6	291
Republic Steel Corp.	4.1	496
International Paper Co.	3.7	433
Continental Oil Co.	3.1	273
Aluminium Limited	0.2	204
Sinclair Oil Corp.	0.0	153
Standard & Poor's Industrial Index	3.7	451

investor who owned $100.00 worth of Sinclair Oil, for example, at the beginning of 1952 and managed it in this way had an investment worth $153.00 at the end of 1961, as is shown at the bottom of Table 13. After deducting his original principal of $100.00, he shows a profit of $53.00. This profit, which may also be expressed as 53 per cent, reflects both market appreciation and dividends paid over the ten-year period. In order to measure the corresponding profit performance of the stock market in general, a profit index was constructed for the Standard & Poor's Industrial Stock Index; this appears at the bottom of Table 13. Stocks with high dividend growth tended to have high profit indexes; and stocks with low dividend growth tended to have low profit indexes. The average dividend growth and the average profit index for the six stocks in each group in Table 13 are presented below, together with the respective figures for the Standard & Poor's Index.

	Group Average	
Group	Dividend Growth Rate	Profit Index
I	11.6%	$933
II	7.9	656
III	6.8	513
IV	5.6	296
V	2.6	308
Standard & Poor's	3.7	451

With the exception of Group IV (which had an average index $12.00 below that of Group V), groups with higher dividend growth had higher profits, and groups with lower dividend growth had lower profits. Group IV's low index is primarily attributable to the relatively low profit index of Amerada Petroleum. Analysis of the stocks in Table 13 supports the conclusion that a definite correlation existed between dividend growth and investment profits during the 10-year period.

Variable Rate Method and Investor Profits

In this section our thirty stocks will be examined to determine whether those stocks which appear most undervalued at the beginning of 1952, according to the variable rate method, had the largest investor profits during the subsequent ten years, 1952-61. The purpose of this analysis is not to determine the exact profit that a typical investor could have made during this period by using the variable rate method, but rather to determine whether that approach is a fundamentally sound and practical method to use in the actual selection of the most profitable investments.

In order to use the variable rate method in making valuations of the thirty stocks at the beginning of 1952, the investor must determine three factors: the initial discount rate, the initial growth rate, and the transitional period.

Initial discount rates

In the selection of discount rates, the Fitch Stock Quality Ratings[3] were used as a guide. The thirty stocks had the following four Fitch designations:

High grade
A - sound

Investment merit
BBB - best of group
BB - good
B - fair

The Fitch ratings are based on the stability of earnings, the safety of dividends, the financial position of the company, the character of the industry, and the position of the company within it.[4] Initial discount rates based on the Fitch ratings, were assigned to the thirty stocks as shown in Table 14. While

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[3] *The Fitch Stock Record,* The Fitch Publishing Co. Inc. (subsequently merged into Standard & Poor's Corp.), Jan. 1, 1958.

[4] Since an A rating (including AAA and AA) is usually applied to preferred stocks, "... an A rating for a common stock is a rarity and would indicate a sound investment. . . . safety of continued dividend payments is anticipated. The BBB rating indicates a moderate investment

TABLE 14

Fitch Quality Ratings and Selected Initial Discount Rates for 30 Stocks

Company	Fitch Quality Rating	Initial Discount Rate (Per Cent)
Best Quality		
DuPont (E. I.)	A	5
General Electric	BBB	5
International Business Machines	BBB	5
Standard Oil (N. J.)	BBB	5
Union Carbide	BBB	5
Good Quality		
Aluminium Ltd.	BB	6
Amerada Petroleum	BB	6
Bethlehem Steel	BB	6
Continental Oil	BB	6
Firestone Tire	BB	6
General Motors	BB	6
Goodrich (B. F.)	BB	6
Goodyear	BB	6
Gulf Oil	BB	6
International Paper	BB	6
Minnesota Mining & Manufacturing	BB	6
National Lead	BB	6
Shell Oil	BB	6
Socony Mobil Oil	BB	6
Standard Oil (Calif.)	BB	6
Texaco	BB	6
U. S. Gypsum	BB	6
U. S. Steel	BB	6
Fair Quality		
Armco Steel	B	$6\frac{1}{2}$
Central & South West	B	$6\frac{1}{2}$
General Public Utilities	B	$6\frac{1}{2}$
Merck	B	$6\frac{1}{2}$
Republic Steel	B	$6\frac{1}{2}$
Sinclair Oil	B	$6\frac{1}{2}$
Texas Utilities	B	$6\frac{1}{2}$
Standard & Poor's Industrial Index		$6\frac{1}{2}$*

*An initial discount rate of $6\frac{1}{2}$ per cent, based on the investment quality of the Standard & Poor's stocks, was previously selected for the Standard & Poor's Index. (See Chap. II, p. 16.)

investors may disagree with the quality ratings assigned at that time to some of the individual stocks, the ratings appear to be reasonable.

Initial growth rates and transitional periods

In the selection of assumed growth rates and transitional periods, the actual dividend growth rates for the thirty stocks during the 1952-62 period were used as a basis for classifying the stocks into three groups, as shown in Table 15.

Group with Favorable Growth.

The first category of Table 15, designated as the Group with Favorable Growth, includes stocks with actual average annual dividend growth rates during the 1952-61 period of between 4.6 per cent and 9.1 per cent (see column 1, Table 15; these rates were previously shown in Table 13). The dividend growth rates in the *terminal period* are shown in column 2; these rates are the actual average growth rates in the 3-year period, 1960-62, which is at the end of the 1952-61 period.[5] Of the 22 stocks in this group, 16 had terminal rates lower than their average rate for the 1952-61 period as a whole (in col. 1). For simplicity, all 22 stocks in this group were assigned an assumed transitional period of ten years (col. 6); consequently, it was assumed that the growth rate in the eleventh year (1962) would be 4 per cent. Next, assumed initial growth rates were selected (col. 3). Initial growth rates were selected for each

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merit. In issues of BB grade, the investment element is not so pronounced, and this is yet more true of the B group. Issues of lower rating must be regarded as relatively less promising, dividendwise. . ." (*Ibid.*)

[5] In identifying the actual growth rate that existed for each stock at the end of the 10-year period (1952-61), an average annual rate of three years—1960, 1961, 1962—was used rather than the growth rate for the single year 1961. The reason for this procedure was to avoid using the dividend declarations of a single year, which might possibly be unrepresentative of the growth trend during that period of time.

stock in such a manner that the *assumed* average growth rate for the whole period, in column 4, would approximate the corresponding *actual* average rate in column 2. This same general approach was applied to the other two groups of stocks in Table 15.

Group with Depressed and Mature Growth

The stocks in the second category of Table 15 were those whose actual average growth rates in the 1952-61 period ranged from 0 per cent to 4.1 per cent (see col. 1). Two of the five stocks in this group had actual rates of 3.7 per cent and 4.1 per cent, very close to 4 per cent; consequently these stocks were considered to have matured and a transitional period of 0 years was assigned to them (col. 6). This means that their growth rates were assumed to remain at a constant 4 per cent in each year.

The other three companies with actual growth rates (in col. 1) considerably below 4 per cent per year were considered to be temporarily depressed. Since Aluminium and Sinclair had actual growth rates of 0.2 per cent and 0 per cent, respectively (col. 1), they were assigned an assumed initial growth rate of 0 per cent and a transitional period of twenty years. This means that their assumed growth rates will gradually rise from 0 per cent in 1952 to 4 per cent by 1972. Selection of an initial growth rate of 2 per cent and a transitional period of ten years for Continental Oil produced an assumed average growth rate in 1952-61 of 2.9 per cent (col. 4) which closely approximates the actual rate of 3.1 per cent (in col. 1).

Group with Very High Growth

The stocks in the third category of Table 15 were those whose actual average growth rates in the 1952-61 period fell between 10.8 per cent and 17.4 per cent (col. 1). Since two of these stocks, International Business Machines and Minnesota Mining, had very high terminal growth rates (30.2 per cent and

TABLE 15

Actual and the Assumed Average Annual Dividend Growth Rates for 30 Stocks, 1952-61

Company	Actual Average Annual Dividend Growth Rate		Assumed Average Annual Dividend Growth Rate			Assumed Transitional Period (Years)
	Whole Period 1952-61 (Per Cent)	Terminal Period 1960-62 (Per Cent)	Initial Growth Rate 1952 (Per Cent)	Whole Period† 1952-61 (Per Cent)	Terminal Period‡ 1960-62 (Per Cent)	
With Favorable Growth						
Amerada	6.0	14.5	8	6.2	4.4	10
Armco	7.2	0.0	10	7.3	4.6	10
Bethlehem Steel	9.1	-3.2*	12	8.4	4.8	10
Central & South West	8.5	6.3	12	8.4	4.8	10
DuPont	7.9	2.3	10	7.3	4.6	10
Firestone	6.3	7.0	8	6.2	4.4	10
General Electric	7.2	0.0	10	7.3	4.6	10
General Motors	6.5	14.5	8	6.2	4.4	10
General Public Utilities	6.4	3.1	8	6.2	4.4	10
Goodrich (B. F.)	5.8	0.0	8	6.2	4.4	10
Goodyear	7.9	6.8	12	8.4	4.8	10
Gulf	8.9	16.4	14	10.5	5.0	10
Merck	7.2	2.9	10	7.3	4.6	10
National Lead	8.8	0.0	12	8.4	4.8	10
Shell Oil	5.1	2.2	6	5.1	4.2	10
Socony Mobil	4.6	5.5	5	4.6	4.1	10
Standard Oil (Calif.)	5.4	1.6	6	5.1	4.2	10
Standard Oil (N. J.)	5.2	3.6	6	5.1	4.2	10
Texaco	8.0	14.0	12	8.4	4.8	10
Union Carbide	6.0	0.0	8	6.2	4.4	10
U. S. Gypsum	7.9	0.0	12	8.4	4.8	10
U. S. Steel	7.2	-2.9*	10	7.3	4.6	10
With Depressed and Mature Growth						
Aluminium Ltd.	0.2	2.9	0	0.9	1.8	20
Continental Oil	3.1	1.0	2	2.9	3.8	10
International Paper	3.7	3.5	4	4.0	4.0	0
Republic Steel	4.1	-5.9*	4	4.0	4.0	0
Sinclair Oil	0.0	-12.7*	0	0.9	1.8	20
With Very High Growth						
International Business Machines	17.4	30.2	20	17.1	14.2	25
Minnesota Mining	14.3	17.0	18	15.5	13.0	25
Texas Utilities	10.8	7.5	14	11.0	8.0	15
Standard & Poor's Industrial Index	3.7	5.0	4	4.0	4.0	0

*Average annual decline.

†Median of the ten assumed annual rates.

‡Median of the three assumed annual rates.

17.0 per cent respectively) in column 2, they were assigned transitional periods of twenty-five years; i.e., their growth rates would gradually decline to 4 per cent by 1977. Since Texas Utilities' terminal growth rate was 7.5 per cent in column 2, a transitional period of fifteen years was assigned to it. Initial growth rates were selected for these three stocks in the same manner as was done for the stocks in the other two categories; that is, the *assumed* average growth rate for the 1952-61 period (col. 4) was selected to approximate the *actual* average rates in column 1.

The present values

All the necessary assumptions have now been quantified in order that variable rate price-dividend ratios may be selected from the tables in Appendix B. Our assumptions consisted of (1) the initial discount rates, set forth in Table 14, (2) the initial growth rates, and (3) the transitional periods, as set forth in columns 3 and 6 of Table 15. The resultant price-dividend ratios for the thirty stocks at the beginning of 1952 (as of December 31, 1951) are shown in Table 16, column 3. Next, each stock's present value, as shown in column 4, was determined by multiplying the stock's price-dividend ratio in column 3 by its appropriate dollar dividend rate in column 1. In order to avoid the possible abnormal dividend results sometimes produced by a single year, the dividend rate used in column 1 was not merely that of 1951, but rather a 3-year arithmetic average of the dividend rates in 1950, 1951, and 1952. These dividend rates are the actual rates paid in that period.

Relative value

The actual closing market prices for the end of 1951 (as reported in the *Wall Street Journal*) are shown in column 2. Finally, each stock's present value was compared to its market price in order to determine its relative value, that is, the relative investment attractiveness of its price at the end of

TABLE 16

Relative Values on December 31, 1951, and Profit Index During the 1951-61 Period for 30 Selected Stocks

Company	Actual		Present Value December 31, 1951		Relative Value December 31, 1951 (Per Cent)	Profit Index 1952-61
	Dividend 1950-52 Average	Market Price December 31, 1951	Price-Dividend Ratio	Price		
					Undervalued	
Group A						
International Business Machines	$4.00	$218.50	182.8	$731.20	-70	$[illegible],391
Bethlehem Steel	4.03	51.50	39.4	158.78	-68	573
U. S. Gypsum	8.17	108.50	39.4	321.90	-66	717
General Motors	4.67	52.00	32.8	153.18	-66	535
U. S. Steel	3.15	39.875	36.0	113.40	-65	615
Armco	3.33	38.375	31.9	106.23	-64	609
Group B						
General Electric	3.22	59.50	48.0	154.56	-62	515
Goodyear	2.83	44.00	39.4	111.50	-60	1,015
Republic Steel	4.08	41.75	24.2	98.74	-58	496
Texaco	3.10	56.125	39.4	122.14	-54	522
DuPont	4.15	92.00	48.0	199.20	-54	365
Minnesota Mining	0.93	48.25	105.0	97.65	-51	953
Group C						
Standard Oil (N. J.)	3.62	75.75	39.7	143.71	-47	307
Central & South West	0.92	17.25	34.9	32.11	-46	703
National Lead	1.40	30.50	39.4	55.16	-45	482
Texas Utilities	1.36	35.00	45.5	61.88	-43	770
General Public Utilities	1.27	21.25	29.1	36.96	-42	493
Union Carbide	2.50	63.625	43.6	109.00	-42	257
Group D						
Gulf	2.00	54.00	43.2	86.40	-38	431
Standard Oil (Calif.)	2.70	50.875	29.9	80.73	-37	369
International Paper	2.83	49.00	27.2	76.98	-36	433
Firestone Tire	3.17	70.25	32.8	103.98	-32	567
Socony Mobil	1.72	35.125	28.5	49.02	-28	291
Shell Oil	3.00	69.00	29.9	89.70	-23	342
Group E						
Goodrich (B. F.)	2.44	61.75	32.8	80.03	-23	314
Continental Oil	2.50	56.25	24.7	61.75	- 9	273
Sinclair Oil	2.53	42.75	17.7	44.78	- 4	153
					Overvalued	
Merck	0.75	32.25	31.9	23.93	+35	360
Aluminium Ltd.	3.67	106.00	19.8	72.67	+46	204
Amerada	3.00	147.50	32.8	98.40	+50	186
Standard & Poor's Industrial Index	1.47	24.23	24.2	35.57	-32	451

1951. The results of this comparison are shown in column 5.[6] The stocks are listed in the order of their price attractiveness; the most deeply undervalued stock is listed first, followed by the less undervalued stocks, in descending order of relative attractiveness. The stock at the bottom of the list (Amerada) was the least attractive, or most overvalued. Similar figures for the Standard & Poor's Industrial Index are shown at the bottom of the table.

In column 6, the profit indexes are shown again for each stock. It is easy to see that a strong positive relation exists between price attractiveness on the one hand and investment profits on the other. Relatively speaking, stocks that were more undervalued had larger profits, and stocks that were less undervalued, or overvalued, had smaller profits during the 10-year period. An even higher correlation would have been achieved if a better fit had been made between the *assumed* dividend payments and the *actual* dividend payments, and if the period studied had been more than ten years.[7]

[6] These figures were computed as follows:

Let V = the stock's present value
M = the stock's actual market price
R = the stock's relative value

Then $\frac{M - V}{V} = R$

Using IBM as an example,
V = \$731.20
M = 218.50

$$\frac{\$218.50 - \$731.20}{\$731.20} = \frac{-\$512.70}{\$731.20} = -.701 \text{ or } -70\%.$$

[7] These points may be illustrated by two stocks, IBM and Bethlehem Steel.

Stock	Relative Value	Profit Index	Terminal Period (1960-62) Growth Rate	
			Actual	Assumed
IBM	-70%	\$2,391	30.2%	14.2%
Beth. Steel	-68	573	-3.0	4.8

In the tabulation above, IBM was undervalued by only two percentage points more than Bethlehem; however, IBM showed considerably more profit in

In general, most of our stocks at the beginning of 1952 were considerably undervalued and correspondingly showed substantial profits in the subsequent ten years. This was generally consistent with the stock market as represented by the Standard & Poor's Index, which was 32 per cent undervalued in 1952 and subsequently had a profit index of 451.00, or a 351 per cent profit in the 1952-61 period.

The stocks in Table 16 are arranged in five groups in the order of their relative values. An average of the relative values (in col. 5) and of the profit indexes (in col. 6) of the six stocks in each group is presented below.

Group		Group Average Relative Value	Group Average Profit Index
A	*under*valued	-66%	$907
B	"	-56	644
C	"	-44	502
S. & P.	"	-32	451
D	"	-32	406
E	*over*valued	+16	248

In this tabulation, the strong correlation between relative values and investment profits becomes obvious. Without exception, the groups with relatively greater undervaluations had the larger profits.

Conclusion

The study of thirty stocks presented in this chapter showed a close relation between the dividend income and the investor profits produced during the 10-year period, 1952-61. While this close relation cannot be guaranteed for the future, it is likely to

the profit indexes. If higher assumed dividend growth rates were used for IBM in the terminal period (1960-62) and in the years beyond 1962, so that the assumed rates were closer to the rates which actually occurred, then IBM would have appeared even more undervalued. Likewise if lower assumed dividend growth rates were used for Bethlehem in the terminal period, etc., so that the assumed rates came closer to the rates which actually occurred, then Bethlehem would have appeared less undervalued.

continue. Furthermore, such a relationship is consistent with the present value theory. To avoid a misinterpretation of the research findings, it may be well to repeat that the objective of this chapter is not to determine the actual profit that a typical investor could have obtained over a 10-year period by the use of the variable rate method, but rather to determine whether the variable rate approach is a fundamentally sound and practical method for selecting the most profitable investments.

CHAPTER VI

CONCLUSIONS

The variable rate method, like all present value methods, has several limitations. The present values provided by such methods are only as accurate as the assumptions used to determine them. Consequently, the most difficult task of appraisal still remains, since the investor must decide what quantities to assign to his assumptions. With the variable rate method, he must decide on three important quantities—the initial growth rate, the initial discount rate, and the transitional period.

The present value approach is probably not very useful to the short-term investor or speculator in forecasting short-term price swings. For the intermediate-term or long-term investor, however, it has considerable merit. Although in the short run, market prices of stocks seem invariably to be above or below their so-called "true" values, over long periods their prices frequently intersect these values so that the investor, in time, has opportunities to adjust his investment positions and thereby take advantage of these market fluctuations. Also, investors who hold diversified portfolios of stocks may show the best results. Since investors occasionally make unrealistic assumptions or miscalculations which result in a loss, such losses will more easily be offset by gains if a number of stocks are held.

The present value method is rather difficult to apply to stocks which currently pay no dividends. The present value approach, including the variable rate method, can be adapted to such cases, and tables can be constructed; however, before the investor can use such a table, he must first estimate when dividends will commence and then estimate the amount of the first dividend.

The variable rate tables presented in this book would not be suitable for those stocks whose future dividends are expected to grow for a time at increasing rather than decreasing rates. Nor would our tables be suited for stocks whose future dividends are expected to grow at high but constant rates year in and year out. Although these cases are not common, tables could be constructed to incorporate such possibilities.

It is sometimes argued that the present value method is useful only to those investors who plan to hold an investment for dividend income for fifty to one hundred years. There are, of course, many institutional investors who do hold investments over several decades; but this method is not by any means limited to them. The present value of an investment reflects not only the estimated future dividends but also the present value of the estimated future principal amount that the investor expects to receive whenever he sells the stock. This principal amount is simply the present value (at the time of sale) of the expected future dividends from that date forth. This assumption is implicit not only in the present value approach but in the variable rate method as well. In order to visualize this point, let us use the IBM illustration previously presented in Chapter III. Let us say that our investor wants to know the present value for IBM on the assumption that the stock will be held for only 17 years instead of 143 years, as was assumed in Table 9 (p. 30). In this instance the present value of IBM would be equal to (1) the dividends to be paid over the next seventeen years, discounted to the present, and (2) the estimated value of one share at the end of the seventeenth year, discounted to the present. Table 17 shows these computations. Column 1 shows the future dividends estimated for IBM, column 2 the discount factors used, and column 3 the present value of dividends expected. The sum of the present values of the seventeen dividends is $47.49881. The estimated value of one share at the end of the seventeenth year is $318.53268, as shown toward the bottom of column 1. The present value of this share of stock appears as $94.76 in column 3. The sum of the present values of the seventeen dividends and of the estimated sale price of one figure was based on a current dividend rate of $1.00 as was Table 9.

TABLE 17

Present Values Computed for International Business Machines Corporation, Assuming That the Investor Sells the Stock at the End of the 17th Year at a Price Equal to Its *Then* Present Value

Future Year	Future Yearly Dividends (as shown in column 1, Table 9)	Yearly Discount Factors (as shown in column 3, Table 8)	Present Values of Future Yearly Dividends (column 1 multiplied by column 2)
(Last year	$1.00000)		
1st	1.25000	.95238095	$1.19047
2nd	1.54500	.90638198	1.40036
3rd	1.88799	.86198923	1.62742
4th	2.28064	.81918612	1.86831
5th	2.72314	.77795360	2.11848
....	....	....	....
13th	7.26730	.50164283	3.64559
14th	7.76148	.47335418	3.67392
15th	8.18060	.44634408	3.65136
16th	8.50782	.42057702	3.57819
17th	8.84813	.39601680	3.50401
Sum of the present values of the 17 dividends			$47.49881
17th (one share of stock)	$318.53268*	.29748916†	$94.76‡
Sum of the present values of dividends and of one share			$142.26

*The expected value of one share of stock at the end of the 17th year is equal to the dividend paid in the 17th year ($8.84813) times 36.0 (the price-dividend ratio for a very high quality stock with an initial discount rate of 5 per cent and an initial growth rate of 4 per cent).

†The discount factor .29748916 is equivalent to $(1 + .0739)^{-17}$ which is formed by using a discount rate of 7.39 per cent. This discount rate is higher than the rate (5.6 per cent) used for the dividend in the 17th year primarily because profits expected by investors from future market price appreciation should be discounted at higher assumed rates of return. The reason for this is that uncertainties concerning dividend projections are subject only to the business risks encountered by the company, while uncertainties concerning a projected future market price are subject to the market risk. For the investor who plans to sell his stock at some future time, the market risk is more serious not only because of the money loss involved but also because changes in market prices are usually more frequent, more volatile, and more uncertain than changes in dividend rates.

‡$94.76 is the present value of one share of stock sold at $318.53 at the end of the 17th year ($94.76 is equal to $318.53268 multiplied by .29748916).

Notice that the present value of \$142.26 and the price-dividend ratio of 142.26 are the same whether IBM is held for seventeen years or 143 years. This is as it should be. In determining the present value of a stock today, it should make no difference *when* a stock is sold as long as it is assumed that it is sold at a price equal to its present value at the time of sale.

The present value approach is sometimes attacked because it relies on a long-term forecast of dividends which, in reality, cannot be perfect. There are several answers to this attack. First, the only fundamental sources of investment value are future income and the subsequent valuations placed on this expected income (except, perhaps, in those few instances where a firm is liquidated).

Second, many other valuation methods indirectly depend on the expectation of future dividends. Some investors prefer to give primary attention to price-earnings ratios and to the outlook for earnings per share rather than to dividends. Earnings of a company are valuable to a stockholder only because they are the principal source of dividends. Some investors make investment decisions which rely on price charts of past security prices. Just as past prices tend to reflect investors' expectations of future dividends and their opinions concerning the uncertainties of these payments, so current price movements tend to reflect changes in dividends now anticipated by investors in the market place. In a way, many investors are unknowingly using the present value concept. Explicitly, however, the present value method requires the investor to formalize or quantify assumptions that he has formerly made in a casual, unconscious, or intuitive manner.

Third, it is precisely because long-term forecasts of dividends are not perfect that our discount rates are increased by the use of variable discount rates over the uncertain future. Also, the present value approach does not require the investor to select and buy stocks at market prices exactly equal to their computed present values. Indeed, the shrewd investor should seek out investments which appear to have present values substantially above current market prices. Thus the investor provides himself with a margin of safety against occasional errors of over-optimism.

It is true that the *application* of the present value approach cannot be perfect. But what alternative valuation method is perfect? The appraisal of investments still remains in large measure an art. As long as investment values depend on a future outcome, there will be no perfect valuation method until men gain a foresight as clear as their hindsight.

The investor who understands the variable rate method knows that this method is no substitute for judgment and experience. Indeed, it requires that the investor use his judgment, background, and common sense in a more precise and rigorous manner than heretofore. The skeptical investor who is not satisfied with any single investment technique may wish to use the present value results as an additional yardstick of comparison against his more subjective analysis and his intuitive "feel" of the market.

APPENDIX A

COMPOUND GROWTH RATE TABLES

The two tables in this appendix make it easy to find the average annual rate of growth in dividends, earnings, or any other series of numerical data for a given 5- or 10-year period.

Let us use IBM as an illustration. What was the annual average rate of growth in dividends during the 5-year period 1958-62? An average of the dividends in 1956, 1957, and 1958, shows that the dividends increased from \$0.97 to \$3.00 in 1962 (see Table 6, p. 25). The dividend rate increased \$2.03—an increase of 2.093 times, or 209.3 per cent during the 5-year period as a whole.

$$\frac{\$2.03}{\$0.97} = 2.093, \text{ or } 209.3\%.$$

Turning to Table A-1, which pertains to 5-year growth periods, we find in the column headed 5-Year Growth that 2.093 falls between 1.932 and 2.176; in the column headed Annual Rate of Growth the latter two figures have corresponding annual growth rates of 24 per cent and 26 per cent respectively. Since 2.093 is closer to 2.176 than to 1.932, we can conclude that the annual growth rate of IBM's dividend was a little below 26 per cent. By interpolation we find that the rate was approximately 25.32 per cent, or 25.3 per cent.

If we wish to find IBM's dividend growth rate for the last ten years, 1953-62, using the average of 1951-53 as the base, we observe that dividends increased from \$0.49 to \$3.00 (see Table 6). This was an increase of \$2.51—an increase of 5.122 times, or 512.2 per cent during the 10-year period as a whole.

$$\frac{\$2.15}{\$0.49} = 5.122, \text{ or } 512.2\%.$$

TABLE A-1

Growth over a 5-Year Period as a Whole and Corresponding Average Annual Compound Rate of Growth during the 5-Year Period

Decrease	
5-Year Decrease	Annual Rate of Decline (Per Cent)
-0.410	-10
-0.376	- 9
-0.341	- 8
-0.304	- 7
-0.266	- 6
-0.226	- 5
-0.185	- 4
-0.143	- 3
-0.096	- 2
-0.049	- 1
0.000	0
Increase	
5-Year Growth	Annual Rate of Growth (Per Cent)
0.025	½
0.051	1
0.077	1½
0.104	2
0.131	2½
0.159	3
0.188	3½
0.217	4
0.246	4½
0.276	5
0.307	5½
0.338	6
0.370	6½
Increase (Cont.)	
5-Year Growth	Annual Rate of Growth (Per Cent)
0.403	7
0.436	7½
0.469	8
0.504	8½
0.539	9
0.611	10
0.685	11
0.762	12
0.842	13
0.925	14
1.011	15
1.100	16
1.288	18
1.488	20
1.703	22
1.932	24
2.176	26
2.436	28
2.713	30
3.484	35
4.378	40
5.410	45
6.594	50
15.413	75
31.000	100

Turning to Table A-2, which pertains to 10-year growth periods, we find under the heading 10-Year Growth that 5.122 lies between 4.234 and 5.192, and consequently falls between an annual growth rate of 18 per cent and one of 20 per cent. Since 5.122 is very close to 5.192, the actual growth rate is a little under 20 per cent. By interpolation we find that the rate was approximately 19.85 per cent, or 19.9 per cent.

These tables may also be used in making a forecast of dividends, or of some other numerical series, five or ten years hence. Assume that a stock is currently earning $2.00 a share and that earnings will grow at 6 per cent per annum for the next five years. Referring to Table A-1, we find that earnings five years hence should be 0.338 times larger, or that earnings are expected to increase by 33.8 per cent. The earnings five years from now should be approximately $2.68.

$$\$2 \times 0.338 = \$0.676 \text{ increase,}$$

$$\$2.000 + \$0.676 = \$2.676, \text{ or } \$2.68.$$

The tables also make it easy to find the average annual rate of decrease in dividends, earnings, or any other series of numerical data for a given 5- or 10-year period. Let us use Armco Steel as an illustration. What was the annual average rate of decrease in earnings during the 5-year period, 1958-62? An average of the earnings in 1956, 1957, and 1958 shows earnings decreasing from $4.88 to $3.10 in 1962 (see Table 10, p. 37). The earnings decreased $1.78—a decrease of 0.365, or 36.5 per cent during the 5-year period as a whole.

$$\frac{\$1.78}{\$4.88} = 0.365, \text{ or } 36.5\% .$$

Turning to Table A-1, which pertains to 5-year periods, we find under the column heading 5-Year Decrease that -0.365 falls between -0.376 and -0.341; under the column heading Annual Rate of Decline the latter two figures have corresponding annual rates of decrease of -9 per cent and -8 per cent respectively. By interpolation we find that the rate of decrease was approximately -8.69 per cent, or -8.7 per cent.

TABLE A-2

Growth over a 10-Year Period as a Whole and Corresponding Average Annual Compound Rate of Growth during the 10-Year Period

Decrease	
10-Year Decrease	Annual Rate of Decline (Per Cent)
-0.651	-10
-0.611	- 9
-0.566	- 8
-0.516	- 7
-0.461	- 6
-0.401	- 5
-0.335	- 4
-0.264	- 3
-0.183	- 2
-0.096	- 1
0.000	0

Increase	
10-Year Growth	Annual Rate of Growth (Per Cent)
0.051	½
0.105	1
0.161	1½
0.219	2
0.280	2½
0.344	3
0.411	3½
0.480	4
0.553	4½
0.629	5
0.708	5½
0.791	6

Increase (Cont.)	
10-Year Growth	Annual Rate of Growth (Per Cent)
0.877	6½
0.967	7
1.061	7½
1.159	8
1.261	8½
1.369	9
1.594	10
1.839	11
2.106	12
2.395	13
2.707	14
3.046	15
3.411	16
4.234	18
5.192	20
6.305	22
7.594	24
9.086	26
10.806	28
12.786	30
19.107	35
27.925	40
40.085	45
56.665	50
268.39	75
1,023.00	100

APPENDIX B

VARIABLE RATE TABLE B-1

2-Year Transitional Period

Price-Dividend Ratios

Initial Growth Rates (Per Cent)	Initial Discount Rates							
	5%	6%	6½%	7%	8%	9%	10%	12%
Depressed								
0	34.0	25.6	22.8	20.5	17.1	14.6	12.7	10.2
1	34.5	26.0	23.1	20.8	17.3	14.8	12.9	10.3
2	35.0	26.4	23.5	21.1	17.6	15.0	13.1	10.5
3	35.5	26.8	23.8	21.4	17.8	15.2	13.3	10.6
Constant								
4	36.0	27.2	24.2	21.7	18.1	15.5	13.5	10.8
High								
5	36.5	27.6	24.5	22.1	18.3	15.7	13.7	10.9
6	37.0	27.9	24.9	22.4	18.6	15.9	13.9	11.1
8	38.1	28.7	25.6	23.0	19.1	16.3	14.3	11.4
10	39.1	29.5	26.3	23.6	19.6	16.8	14.7	11.7
12	40.2	30.3	27.0	24.3	20.2	17.3	15.1	12.0
14	41.3	31.2	27.7	24.9	20.7	17.7	15.5	12.3
16	42.4	32.0	28.4	25.6	21.3	18.2	15.9	12.6
18	43.5	32.8	29.2	26.3	21.8	18.6	16.3	12.9
20	44.6	33.7	29.9	26.9	22.4	19.1	16.7	13.3
25	47.5	35.8	31.9	28.7	23.8	20.3	17.7	14.1
30	50.5	38.1	33.8	30.4	25.3	21.6	18.8	15.0
35	53.5	40.3	35.8	32.2	26.8	22.9	19.9	15.9
40	56.6	42.7	37.9	34.1	28.3	24.2	21.1	16.8
50	63.1	47.5	42.2	38.0	31.5	26.9	23.5	18.6
60	69.9	52.6	46.8	42.0	34.9	29.8	26.0	20.6
70	77.0	58.0	51.5	46.3	38.4	32.8	28.6	22.7

VARIABLE RATE TABLE B-2

4—Year Transitional Period

Price-Dividend Ratios

Initial Growth Rates (Per Cent)	Initial Discount Rates							
	5%	6%	6½%	7%	8%	9%	10%	12%
Depressed								
0	32.7	24.7	22.0	19.8	16.5	14.1	12.3	9.8
1	33.5	25.3	22.5	20.3	16.9	14.4	12.6	10.1
2	34.3	25.9	23.1	20.8	17.3	14.8	12.9	10.3
3	35.2	26.5	23.6	21.2	17.7	15.1	13.2	10.5
Constant								
4	36.0	27.2	24.2	21.7	18.1	15.5	13.5	10.8
High								
5	36.8	27.8	24.7	22.2	18.5	15.8	13.8	11.0
6	37.7	28.5	25.3	22.8	18.9	16.2	14.1	11.2
8	39.5	29.8	26.5	23.8	19.8	16.9	14.7	11.7
10	41.3	31.1	27.7	24.9	20.7	17.7	15.4	12.2
12	43.2	32.5	28.9	26.0	21.6	18.4	16.1	12.8
14	45.1	34.0	30.2	27.2	22.5	19.2	16.8	13.3
16	47.1	35.5	31.5	28.3	23.5	20.1	17.5	13.9
18	49.2	37.0	32.9	29.6	24.5	20.9	18.2	14.4
20	51.3	38.6	34.3	30.8	25.6	21.8	19.0	15.0
25	56.9	42.8	38.0	34.1	28.3	24.1	21.0	16.6
30	62.9	47.3	42.0	37.7	31.2	26.6	23.1	18.3
35	69.4	52.1	46.2	41.5	34.3	29.2	25.4	20.0
40	76.3	57.3	50.8	45.6	37.7	32.1	27.8	21.9
50	91.7	68.7	60.9	54.6	45.1	38.3	33.2	26.1
60	109.1	81.7	72.4	64.8	53.5	45.4	39.3	30.9
70	128.7	96.3	85.3	76.4	63.0	53.4	46.2	36.2

VARIABLE RATE TABLE B-3

5-Year Transitional Period

Price-Dividend Ratios

Initial Growth Rates (Per Cent)	Initial Discount Rates							
	5%	6%	6½%	7%	8%	9%	10%	12%
Depressed								
0	32.2	24.3	21.6	19.5	16.2	13.9	12.1	9.7
1	33.1	25.0	22.2	20.0	16.7	14.3	12.5	9.9
2	34.0	25.7	22.9	20.6	17.1	14.7	12.8	10.2
3	35.0	26.4	23.5	21.2	17.6	15.1	13.1	10.5
Constant								
4	36.0	27.2	24.2	21.7	18.1	15.5	13.5	10.8
High								
5	37.0	27.9	24.8	22.3	18.6	15.9	13.9	11.0
6	38.0	28.7	25.5	22.9	19.1	16.3	14.2	11.3
8	40.2	30.3	26.9	24.2	20.1	17.2	15.0	11.9
10	42.4	31.9	28.4	25.5	21.2	18.1	15.8	12.5
12	44.7	33.7	29.9	26.9	22.3	19.0	16.6	13.1
14	47.1	35.5	31.5	28.3	23.5	20.0	17.4	13.8
16	49.6	37.3	33.1	29.8	24.7	21.0	18.3	14.5
18	52.2	39.3	34.8	31.3	25.9	22.1	19.2	15.2
20	54.9	41.3	36.6	32.9	27.2	23.2	20.2	15.9
25	62.2	46.7	41.4	37.1	30.7	26.1	22.7	17.9
30	70.1	52.6	46.6	41.8	34.5	29.3	25.4	20.0
35	78.8	59.1	52.3	46.9	38.7	32.8	28.5	22.3
40	88.4	66.1	58.6	52.5	43.3	36.7	31.8	24.9
50	110.1	82.2	72.8	65.1	53.6	45.4	39.2	30.6
60	135.7	101.2	89.5	80.0	65.8	55.6	48.0	37.4
70	165.7	123.4	109.0	97.4	80.0	67.5	58.2	45.2

VARIABLE RATE TABLE B-4

6–Year Transitional Period

Price-Dividend Ratios

Initial Growth Rates (Per Cent)	Initial Discount Rates 5%	6%	6½%	7%	8%	9%	10%	12%
Depressed								
0	31.6	23.9	21.3	19.2	16.0	13.7	12.0	9.6
1	32.7	24.7	22.0	19.8	16.5	14.1	12.3	9.8
2	33.7	25.5	22.7	20.4	17.0	14.5	12.7	10.1
3	34.9	26.3	23.4	21.1	17.5	15.0	13.1	10.4
Constant								
4	36.0	27.2	24.2	21.7	18.1	15.5	13.5	10.8
High								
5	37.2	28.0	24.9	22.4	18.6	15.9	13.9	11.1
6	38.4	28.9	25.7	23.1	19.2	16.4	14.3	11.4
8	40.9	30.8	27.4	24.6	20.4	17.4	15.2	12.1
10	43.5	32.7	29.1	26.1	21.7	18.5	16.1	12.8
12	46.2	34.8	30.9	27.7	23.0	19.6	17.1	13.5
14	49.1	36.9	32.8	29.4	24.4	20.8	18.1	14.3
16	52.2	39.2	34.8	31.2	25.8	22.0	19.1	15.1
18	55.4	41.6	36.9	33.1	27.4	23.3	20.2	15.9
20	58.7	44.1	39.1	35.0	29.0	24.6	21.4	16.8
25	67.8	50.8	45.0	40.3	33.3	28.2	24.5	19.2
30	78.0	58.3	51.6	46.2	38.1	32.3	27.9	21.9
35	89.4	66.8	59.1	52.9	43.5	36.8	31.8	24.8
40	102.1	76.2	67.3	60.2	49.5	41.8	36.1	28.1
50	132.0	98.2	86.7	77.4	63.5	53.5	46.1	35.7
60	168.5	125.1	110.3	98.5	80.6	67.8	58.3	45.0
70	212.8	157.7	139.0	123.9	101.2	85.0	72.9	56.1

VARIABLE RATE TABLE B-5

8-Year Transitional Period

Price-Dividend Ratios

Initial Growth Rates (Per Cent)	Initial Discount Rates							
	5%	6%	6½%	7%	8%	9%	10%	12%
Depressed								
0	30.6	23.1	20.6	18.6	15.5	13.3	11.6	9.3
1	31.8	24.1	21.5	19.3	16.1	13.8	12.1	9.7
2	33.2	25.1	22.3	20.1	16.7	14.3	12.5	10.0
3	34.6	26.1	23.2	20.9	17.4	14.9	13.0	10.4
Constant								
4	36.0	27.2	24.2	21.7	18.1	15.5	13.5	10.8
High								
5	37.5	28.3	25.1	22.6	18.8	16.1	14.0	11.1
6	39.0	29.4	26.1	23.5	19.5	16.7	14.5	11.5
8	42.3	31.8	28.2	25.4	21.0	17.9	15.6	12.4
10	45.7	34.4	30.5	27.4	22.7	19.3	16.8	13.3
12	49.4	37.1	32.9	29.5	24.4	20.8	18.0	14.2
14	53.4	40.0	35.5	31.8	26.3	22.3	19.3	15.2
16	57.6	43.1	38.2	34.2	28.2	24.0	20.8	16.3
18	62.1	46.5	41.1	36.8	30.3	25.7	22.3	17.4
20	67.0	50.0	44.2	39.6	32.6	27.6	23.8	18.6
25	80.5	59.9	52.9	47.3	38.8	32.8	28.3	22.0
30	96.2	71.5	63.1	56.3	46.1	38.8	33.4	25.9
35	114.6	84.9	74.8	66.7	54.5	45.8	39.3	30.3
40	135.8	100.5	88.5	78.8	64.3	53.9	46.2	35.5
50	188.8	139.1	122.3	108.7	88.4	73.8	63.0	48.1
60	258.5	190.0	166.7	148.0	119.9	99.9	85.0	64.5
70	349.3	256.0	224.4	198.9	160.8	133.6	113.4	85.5

VARIABLE RATE TABLE B-6

10-Year Transitional Period

Price-Dividend Ratios

Initial Growth Rates (Per Cent)	Initial Discount Rates							
	5%	6%	6½%	7%	8%	9%	10%	12%
Depressed								
0	29.6	22.4	20.0	18.1	15.1	13.0	11.4	9.1
1	31.1	23.6	21.0	18.9	15.8	13.5	11.9	9.5
2	32.6	24.7	22.0	19.8	16.5	14.2	12.4	9.9
3	34.3	25.9	23.1	20.8	17.3	14.8	12.9	10.3
Constant								
4	36.0	27.2	24.2	21.7	18.1	15.5	13.5	10.8
High								
5	37.8	28.5	25.3	22.8	18.9	16.2	14.1	11.2
6	39.7	29.9	26.5	23.8	19.8	16.9	14.7	11.7
8	43.6	32.8	29.1	26.1	21.6	18.4	16.0	12.7
10	48.0	36.0	31.9	28.6	23.6	20.1	17.4	13.7
12	52.7	39.4	34.9	31.3	25.8	21.9	19.0	14.9
14	57.9	43.2	38.2	34.2	28.2	23.8	20.6	16.1
16	63.4	47.3	41.8	37.4	30.7	26.0	22.4	17.5
18	69.5	51.7	45.7	40.8	33.5	28.2	24.3	18.9
20	76.1	56.5	49.9	44.5	36.5	30.7	26.4	20.5
25	95.1	70.3	61.9	55.2	45.0	37.8	32.4	24.9
30	118.2	87.1	76.6	68.1	55.4	46.3	39.6	30.3
35	146.1	107.4	94.3	83.7	67.8	56.6	48.2	36.6
40	179.9	131.8	115.5	102.4	82.8	68.8	58.5	44.2
50	268.7	196.0	171.3	151.6	121.9	100.9	85.3	63.8
60	394.8	286.7	250.2	220.8	176.9	145.8	122.7	91.1
70	570.7	413.2	359.8	317.1	253.1	207.9	174.4	128.6

VARIABLE RATE TABLE B-7

12-Year Transitional Period

Price-Dividend Ratios

Initial Growth Rates (Per Cent)	Initial Discount Rates							
	5%	6%	6½%	7%	8%	9%	10%	12%
Depressed								
0	28.7	21.8	19.5	17.6	14.7	12.7	11.1	8.9
1	30.4	23.0	20.6	18.5	15.5	13.3	11.7	9.4
2	32.1	24.4	21.7	19.6	16.3	14.0	12.2	9.8
3	34.0	25.7	22.9	20.6	17.2	14.7	12.9	10.3
Constant								
4	36.0	27.2	24.2	21.7	18.1	15.5	13.5	10.8
High								
5	38.1	28.7	25.5	22.9	19.0	16.3	14.2	11.3
6	40.3	30.3	26.9	24.2	20.0	17.1	14.9	11.8
8	45.0	33.8	29.9	26.8	22.2	18.9	16.4	12.9
10	50.3	37.6	33.3	29.8	24.6	20.8	18.0	14.2
12	56.1	41.8	37.0	33.1	27.2	23.0	19.9	15.5
14	62.6	46.5	41.1	36.7	30.1	25.4	21.9	17.0
16	69.7	51.7	45.6	40.7	33.3	28.0	24.1	18.6
18	77.5	57.4	50.5	45.0	36.7	30.9	26.5	20.4
20	86.2	63.6	56.0	49.8	40.6	34.0	29.1	22.4
25	112.0	82.2	72.1	64.0	51.9	43.3	36.9	28.0
30	144.6	105.7	92.5	81.9	66.1	54.8	46.5	35.1
35	185.7	135.2	118.1	104.4	83.8	69.3	58.5	43.8
40	237.2	172.0	150.0	132.3	105.9	87.2	73.4	54.5
50	381.2	274.7	238.8	210.0	167.0	136.7	114.3	83.9
60	600.6	430.7	373.4	327.6	259.2	211.0	175.6	127.5
70	929.2	663.6	574.1	502.6	395.9	321.0	265.9	191.4

VARIABLE RATE TABLE B-8

15-Year Transitional Period

Price-Dividend Ratios

Initial Growth Rates (Per Cent)	Initial Discount Rates							
	5%	6%	6½%	7%	8%	9%	10%	12%
Depressed								
0	27.5	21.0	18.7	16.9	14.2	12.3	10.8	8.7
1	29.4	22.4	20.0	18.0	15.1	13.0	11.4	9.2
2	31.4	23.9	21.3	19.2	16.0	13.8	12.1	9.7
3	33.6	25.5	22.7	20.4	17.0	14.6	12.8	10.2
Constant								
4	36.0	27.2	24.2	21.7	18.1	15.5	13.5	10.8
High								
5	38.5	29.0	25.8	23.1	19.2	16.4	14.3	11.3
6	41.2	30.9	27.4	24.6	20.4	17.4	15.1	11.9
8	47.1	35.2	31.2	27.9	23.0	19.5	16.9	13.3
10	53.8	40.0	35.4	31.6	25.9	21.9	18.9	14.8
12	61.4	45.5	40.1	35.8	29.3	24.6	21.2	16.4
14	70.0	51.7	45.5	40.5	33.0	27.7	23.7	18.3
16	79.8	58.7	51.6	45.8	37.2	31.1	26.6	20.3
18	90.9	66.6	58.4	51.8	41.9	34.9	29.8	22.6
20	103.4	75.6	66.1	58.6	47.2	39.2	33.3	25.2
25	142.3	103.1	89.9	79.4	63.6	52.4	44.2	33.0
30	194.5	140.1	121.8	107.1	85.2	69.9	58.5	43.2
35	264.4	189.4	164.1	143.9	113.9	92.8	77.3	56.4
40	357.1	254.5	220.1	192.5	151.6	122.9	101.9	73.6
50	640.0	452.5	389.6	339.4	265.0	213.1	175.1	124.4
60	1,120.6	787.3	675.6	586.6	455.0	363.3	296.6	207.7
70	1,919.6	1,341.5	1,148.1	994.3	767.0	609.0	494.4	342.3

VARIABLE RATE TABLE B-9

20-Year Transitional Period

Price-Dividend Ratios

Initial Growth Rates (Per Cent)	Initial Discount Rates							
	5%	6%	6½%	7%	8%	9%	10%	12%
Depressed								
0	25.7	19.8	17.7	16.1	13.6	11.8	10.4	8.5
1	28.0	21.4	19.2	17.3	14.6	12.6	11.1	9.0
2	30.4	23.2	20.7	18.7	15.7	13.5	11.8	9.5
3	33.1	25.1	22.4	20.2	16.8	14.4	12.6	10.1
Constant								
4	36.0	27.2	24.2	21.7	18.1	15.5	13.5	10.8
High								
5	39.2	29.4	26.1	23.5	19.4	16.6	14.4	11.4
6	42.6	31.9	28.3	25.3	20.9	17.8	15.4	12.2
8	50.5	37.5	33.1	29.5	24.2	20.5	17.6	13.8
10	59.8	44.1	38.8	34.5	28.1	23.6	20.2	15.6
12	70.7	51.8	45.4	40.3	32.6	27.2	23.2	17.7
14	83.7	60.9	53.3	47.1	37.9	31.5	26.7	20.2
16	99.1	71.7	62.5	55.1	44.1	36.4	30.7	23.0
18	117.1	84.3	73.2	64.4	51.3	42.1	35.4	26.3
20	138.4	99.1	85.9	75.3	59.7	48.8	40.8	30.0
25	209.3	148.1	127.7	111.4	87.3	70.5	58.3	42.0
30	314.7	220.5	189.1	164.2	127.4	101.9	83.5	59.1
35	470.0	326.6	278.9	241.1	185.5	147.2	119.6	83.2
40	696.8	480.9	409.2	352.5	269.3	212.0	170.9	117.2
50	1,498.7	1,022.7	865.2	740.9	559.2	435.1	346.5	231.7
60	3,130.4	2,117.8	1,783.7	1,520.5	1,137.1	876.3	691.1	453.2
70	6,356.1	4,270.7	3,584.3	3,044.5	2,260.3	1,728.8	1,353.1	873.2

VARIABLE RATE TABLE B-10

25-Year Transitional Period

Price-Dividend Ratios

Initial Growth Rates (Per Cent)	Initial Discount Rates							
	5%	6%	6½%	7%	8%	9%	10%	12%
Depressed								
0	24.3	18.8	17.0	15.4	13.1	11.4	10.1	8.3
1	26.8	20.6	18.5	16.8	14.2	12.3	10.9	8.8
2	29.5	22.6	20.2	18.3	15.4	13.3	11.7	9.4
3	32.6	24.8	22.1	19.9	16.7	14.3	12.5	10.1
Constant								
4	36.0	27.2	24.2	21.7	18.1	15.5	13.5	10.8
High								
5	39.8	29.8	26.5	23.7	19.6	16.7	14.5	11.5
6	44.0	32.8	29.0	25.9	21.4	18.1	15.7	12.3
8	53.8	39.7	34.9	31.1	25.3	21.3	18.3	14.2
10	65.9	48.1	42.1	37.3	30.1	25.1	22.4	16.3
12	80.7	58.3	50.8	44.8	35.9	29.7	25.1	18.9
14	99.0	70.9	61.5	54.0	42.9	35.2	29.5	21.9
16	121.5	86.3	74.5	65.2	51.4	41.8	34.8	25.5
18	149.1	105.0	90.3	78.7	61.6	49.7	41.2	29.8
20	182.8	127.9	109.6	95.2	73.9	59.3	48.7	34.9
25	303.8	209.1	177.8	153.2	117.1	92.5	74.9	52.0
30	502.1	341.1	288.1	246.5	186.0	144.9	115.8	78.4
35	823.9	553.7	465.1	395.7	205.2	227.3	179.6	118.7
40	1,341.2	893.1	746.7	632.2	467.1	356.2	278.6	180.5
50	3,463.0	2,273.0	1,886.4	1,585.1	1,153.0	865.5	665.9	417.4
60	8,630.6	5,603.9	4,625.0	3,864.1	2,778.1	2,060.4	1,565.7	956.4
70	20,775.4	13,376.7	10,992.2	9,143.2	6,513.8	4,785.5	3,601.4	2,155.9

VARIABLE RATE TABLE B-11

30-Year Transitional Period

Price-Dividend Ratios

Initial Growth Rates (Per Cent)	Initial Discount Rates							
	5%	6%	6½%	7%	8%	9%	10%	12%
Depressed								
0	23.2	18.1	16.3	14.9	12.7	11.1	9.9	8.2
1	25.8	20.0	18.0	16.4	13.9	12.1	10.7	8.7
2	28.8	22.1	19.8	18.0	15.1	13.1	11.5	9.3
3	32.2	24.5	21.9	19.7	16.5	14.2	12.5	10.0
Constant								
4	36.0	27.2	24.2	21.7	18.1	15.5	13.5	10.8
High								
5	40.3	30.2	26.7	24.0	19.8	16.8	14.6	11.6
6	45.2	33.6	29.6	26.5	21.7	18.4	15.9	12.4
8	57.0	41.7	36.6	32.4	26.3	22.0	18.8	14.5
10	72.1	52.0	45.3	39.9	32.0	26.4	22.4	16.9
12	91.4	65.0	56.3	49.3	39.1	31.9	26.8	19.9
14	116.0	81.6	70.2	61.1	47.9	38.8	32.2	23.5
16	147.5	102.6	87.7	76.0	58.9	47.2	38.8	27.9
18	187.6	129.1	109.9	94.7	72.7	57.7	47.0	33.2
20	238.8	162.7	137.8	118.3	89.9	70.8	57.2	39.7
25	435.7	290.9	243.9	207.1	154.3	118.9	94.2	62.9
30	791.3	519.7	431.9	363.7	266.3	201.7	157.0	101.4
35	1,426.9	924.5	763.0	637.9	460.3	343.5	263.4	165.1
40	2,550.3	1,633.7	1,340.5	1,114.1	794.1	585.4	443.2	270.6
50	7,906.4	4,977.1	4,046.8	3,332.3	2,330.3	1,684.0	1,249.3	731.3
60	23,511.2	14,610.2	11,799.8	9,649.9	6,652.9	4,737.5	3,461.2	1,962.5
70	67,091.2	41,277.4	33,164.9	26,978.6	18,397.1	12,951.9	9,351.6	5,172.4

APPENDIX C

VARIABLE RATE TABLE EQUATIONS

The following equations were used in computing the Variable Rate Tables in Appendix B (see Chaps. II, III, and IV for actual illustrations):

Let: G_1 = initial growth rate
G_2 = growth rate in the second year
G_3 = growth rate in the third year
G_4 = etc.
A_1 = dividend amount in the first year
A_2 = dividend amount in the second year
A_3 = etc.

Then

$$A_1 = \$1.00\,(1+G_1)$$
$$A_2 = A_1\,(1+G_2)$$
$$A_3 = A_2\,(1+G_3)$$
$$A_4 = etc.$$

Let: t = the number of transitional years plus one year

Then:

$$G_t = .04$$

Let: E = the percentage point decrease in each succeeding year's growth rate during the transitional period.

Then:

$$\frac{G_1 - G_t}{t - 1} = E$$

$$G_2 = G_1 - E$$
$$G_3 = G_2 - E$$
$$G_4 = G_3 - E$$
$$G_5 = \text{etc.}$$

Let:

D_1 = initial discount rate
D_2 = discount rate in the second year
D_3 = discount rate in the third year
D_4 = etc.

Each succeeding year's future discount rate increases by a constant absolute amount, which is 0.0075 times the initial discount rate, D_1.

Therefore:

$$D_2 = D_1 + .0075\,D_1$$
$$D_3 = D_2 + .0075\,D_1$$
$$D_4 = D_3 + .0075\,D_1$$
$$D_5 = \text{etc.}$$

Let:

n = the last future year computed
R = price-dividend ratio at the present time
R_n = price-dividend ratio at the last future year computed
V_n = value of one share in the last future year computed.

Then: $V_n = R_n A_n$

$$\frac{A_n}{(1+D_n)^n} < .001$$

$$R = \frac{A_1}{(1+D_1)^1} + \frac{A_2}{(1+D_2)^2} + \frac{A_3}{(1+D_3)^3} + \ldots + \frac{A_n}{(1+D_n)^n} + \frac{V_n}{(1+D_n)^n},$$

$$R = \sum \frac{A_i}{(1+D_i)^i} + \frac{V_n}{(1+D_n)^n}$$

APPENDIX D

LONG-TERM HISTORICAL VARIABLE RATE TABLES

As described on page 13, footnote 5, the average yield for a large group of stocks during the 1871-1958 period approximated 5 per cent or a price-dividend ratio of approximately 20.

The tables in this appendix were constructed on the following assumptions: The average stock has a constant growth rate of 4 per cent, an initial discount rate of 6 per cent, and an average price-dividend ratio of about 20. A price-dividend ratio of 19.6 is derived from these assumptions when each succeeding future discount rate is increased by .02 times the initial discount rate (of 6 per cent). The figure 19.6 appears in the appropriate place in Tables D-1, D-2, and D-3. This combination of assumptions was subsequently used to derive the other price-dividend ratios which appear in these tables.

Therefore the only difference between the tables in Appendix B and the tables in this appendix is that the former use future discount rates which increase by .0075 times the initial discount rate, while these use future discount rates which increase by .02 times the initial discount rate.

That is:
$$\begin{aligned} D_2 &= D_1 + .02\,D_1 \\ D_3 &= D_2 + .02\,D_1 \\ D_4 &= \text{etc.} \end{aligned}$$

Since the future discount rates used in this appendix increase more rapidly than those in Appendix B, the price-dividend ratios here are lower and consequently more conservative.

VARIABLE RATE TABLE D-1

5-Year Transitional Period

Price-Dividend Ratios

Initial Growth Rates (Per Cent)	Initial Discount Rates						
	5%	6%	7%	8%	9%	10%	12%
4(Constant rate)	24.4	19.6	16.4	14.1	12.4	11.0	9.1
5	25.0	20.1	16.8	14.5	12.7	11.3	9.3
6	25.7	20.7	17.3	14.9	13.0	11.6	9.5
8	27.1	21.8	18.2	15.7	13.7	12.2	10.0
10	28.6	23.0	19.2	16.5	14.4	12.8	10.5
12	30.1	24.2	20.2	17.3	15.2	13.5	11.0
14	31.7	25.5	21.2	18.2	15.9	14.2	11.6
16	33.4	26.8	22.3	19.1	16.7	14.9	12.1
18	35.1	28.1	23.4	20.1	17.6	15.6	12.7
20	36.9	29.6	24.6	21.1	18.4	16.3	13.3
25	41.7	33.4	27.7	23.7	20.7	18.3	14.9
30	47.0	37.5	31.2	26.6	23.2	20.5	16.7
35	52.7	42.1	34.9	29.8	25.9	22.9	18.6
40	59.0	47.0	39.0	33.2	28.9	25.5	20.6
50	73.3	58.3	48.2	41.0	35.6	31.4	25.3
60	90.1	71.6	59.1	50.2	43.5	38.3	30.8
70	109.8	87.1	71.8	60.9	52.7	46.3	37.1

VARIABLE RATE TABLE D-2

15-Year Transitional Period

Price-Dividend Ratios

Initial Growth Rates (Per Cent)	Initial Discount Rates						
	5%	6%	7%	8%	9%	10%	12%
4(Constant rate)	24.4	19.6	16.4	14.1	12.4	11.0	9.1
5	26.0	20.8	17.4	14.9	13.1	11.6	9.5
6	27.6	22.1	18.4	15.8	13.8	12.2	10.0
8	31.3	25.0	20.7	17.6	15.4	13.6	11.0
10	35.5	28.2	23.2	19.7	17.1	15.1	12.2
12	40.3	31.8	26.1	22.1	19.1	16.8	13.4
14	45.6	35.8	29.3	24.7	21.3	18.6	14.8
16	51.7	40.4	33.0	27.7	23.7	20.7	16.4
18	58.5	45.6	37.0	31.0	26.5	23.0	18.1
20	66.2	51.4	41.6	34.7	29.5	25.6	20.0
25	89.8	69.1	55.5	45.9	38.8	33.4	25.7
30	121.4	92.7	73.9	60.7	50.9	43.5	33.1
35	163.3	123.9	98.1	80.0	66.8	56.7	42.7
40	218.6	164.9	129.8	105.3	87.3	73.8	54.9
50	385.9	288.2	224.5	180.3	148.2	124.0	90.5
60	667.4	494.5	382.2	304.5	248.2	205.9	147.9
70	1,132.0	833.1	639.7	506.2	409.8	337.8	239.3

VARIABLE RATE TABLE D-3

25-Year Transitional Period

Price-Dividend Ratios

Initial Growth Rates (Per Cent)	Initial Discount Rates						
	5%	6%	7%	8%	9%	10%	12%
4 (Constant rate)	24.4	19.6	16.4	14.1	12.4	11.0	9.1
5	26.6	21.3	17.7	15.2	13.3	11.8	9.6
6	29.1	23.2	19.2	16.3	14.2	12.6	10.2
8	34.9	27.4	22.5	19.0	16.4	14.4	11.6
10	41.9	32.6	26.5	22.2	19.0	16.6	13.1
12	50.5	38.8	31.2	25.9	22.0	19.1	14.9
14	60.9	46.4	36.9	30.4	25.6	22.0	17.0
16	73.5	55.4	43.8	35.7	29.9	25.5	19.5
18	88.8	66.4	52.0	42.1	34.9	29.6	22.3
20	107.3	79.6	61.8	49.7	40.9	34.5	25.6
25	172.7	125.6	95.8	75.6	61.3	50.8	36.6
30	277.8	198.8	149.1	115.8	92.5	75.5	53.0
35	445.5	314.2	232.4	178.0	140.2	112.9	77.2
40	711.0	495.4	361.9	273.9	213.0	169.4	113.1
50	1,777.9	1,214.6	869.7	644.8	491.4	382.9	245.5
60	4,322.4	2,908.5	2,050.4	1,496.3	1,122.0	806.2	533.4
70	10,201.4	6,782.6	4,722.9	3,403.1	2,518.9	1,905.7	1,150.3

Date Due

Due	Returned	Due	Returned
JUL 1 4 '66	ML JUL 14 '66		

650.5
M624r
no.42

BUSINESS
ADM.

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650.5M624r no.42
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